



# Representations and Algorithms for Interactive Relighting

Nick Michiels

Promoter: Prof. Dr. Philippe Bekaert

# Representations and Algorithms for Interactive Relighting



[Matterport, 2016]

**Real Estate**



[Hilsmann et al., Eurographics 2013]

**Clothing Industry**



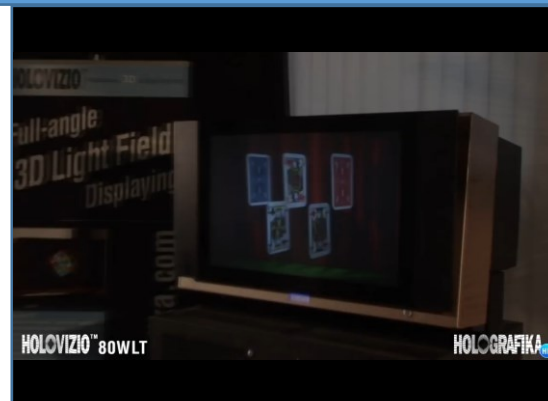
[Microsoft HoloLens, 2016]

**Augmented Reality**

# Representations and Algorithms for Interactive Relighting



immersive experience  
view-dependent



# Representations and Algorithms for Interactive Relighting



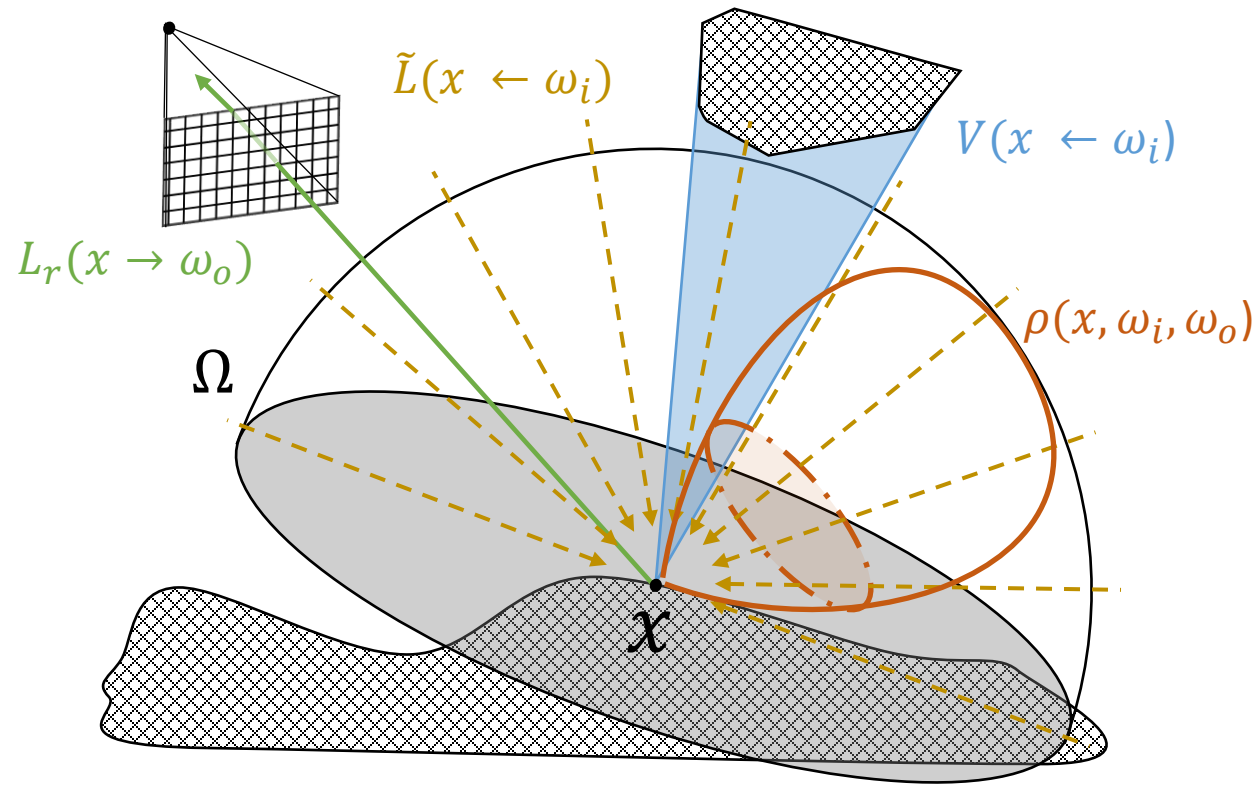
# Representations and Algorithms for Interactive Relighting



# Representations and Algorithms for Interactive Relighting



# Representations and Algorithms for Interactive Relighting

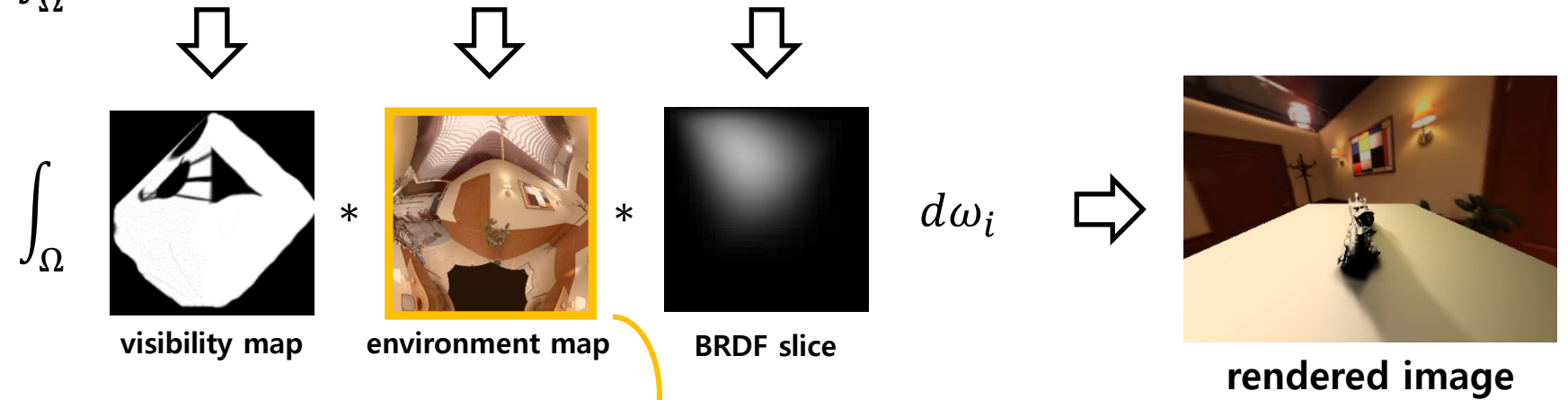


$$L_r(x \rightarrow \omega_0) = \int_{\Omega} V(x \leftarrow \omega_i) * \tilde{L}(x \leftarrow \omega_i) * \tilde{\rho}(x, \omega_i, \omega_0) d\omega_i$$

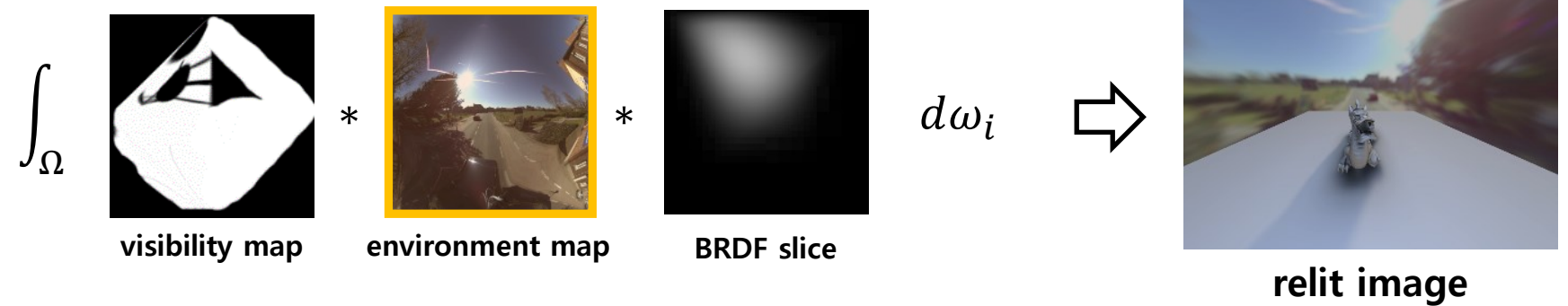
# Representations and Algorithms for Interactive Relighting

$$L_r(x \rightarrow \omega_o) = \int_{\Omega} V(x \leftarrow \omega_i) * \tilde{L}(x \leftarrow \omega_i) * \tilde{\rho}(x, \omega_i, \omega_o) d\omega_i$$

Forward  
Rendering



Relighting





# Representations and Algorithms for Interactive Relighting

## 1. Relighting of **Virtual Objects**



## 2. Relighting of **Real Objects**



1. Texture-illumination ambiguity
2. Simulation of light propagation

# Triple Product Integral

$$L_r(x \rightarrow \omega_o) = \int_{\Omega} V(x \leftarrow \omega) * \tilde{L}(x \leftarrow \omega) * \tilde{\rho}(x, \omega, \omega_o) d\omega$$

pixel domain

$$= \int_{\Omega} \text{[Mask]} * \text{[Image]} * \text{[Kernel]} d\omega$$

new basis representation

$$= \int_{\Omega} \sum_i V_i \Psi_i(\omega) * \sum_j L_j \Psi_j(\omega) * \sum_k \tilde{\rho}_k \Psi_k(\omega) d\omega$$

triple product binding coefficients

$$= \sum_i \sum_j \sum_k V_i \tilde{L}_j \tilde{\rho}_k C_{ijk} \quad \boxed{C_{ijk} = \int_{\Omega} \Psi_i(\omega) \Psi_j(\omega) \Psi_k(\omega) d\omega}$$

# Spherical Harmonics

---

- Original approach [*Sloan et al., 2002*]
- Equivalent of Fourier series on the sphere
- Set of orthogonal functions
- Linear combination of sine and cosine waves

+ Efficient

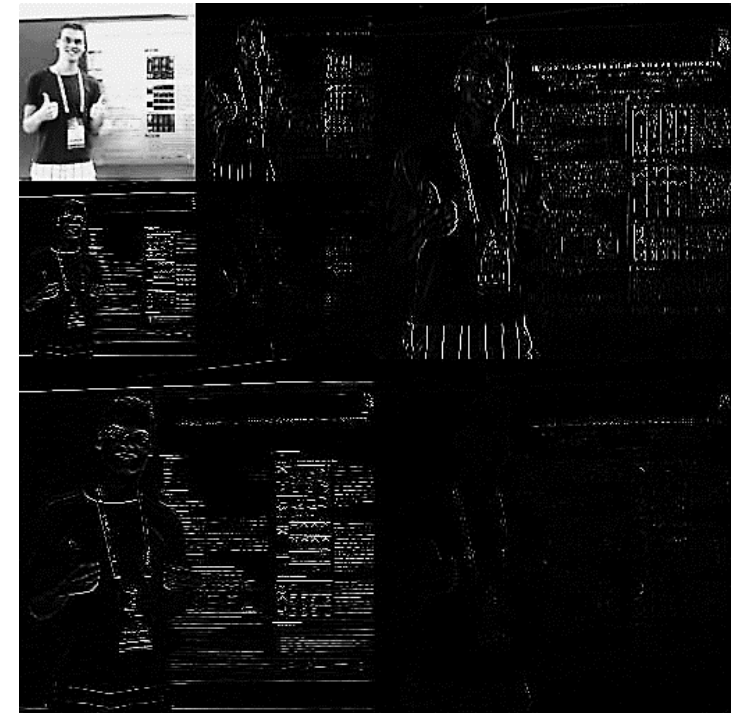
- Low-frequency lighting effects only

# 2D Haar Wavelets

- Haar Tripling Coefficient Theorem [Ng et al., 2004]
- Piecewise constant functions
- Orthonormal basis

$$\Psi_{01}(x, y) = \begin{array}{|c|c|} \hline \blacksquare & \blacksquare \\ \hline \square & \square \\ \hline \end{array} \quad \Psi_{10}(x, y) = \begin{array}{|c|c|} \hline \blacksquare & \square \\ \hline \blacksquare & \square \\ \hline \end{array} \quad \Psi_{11}(x, y) = \begin{array}{|c|c|} \hline \blacksquare & \square \\ \hline \square & \blacksquare \\ \hline \end{array}$$

- + Few coefficients
- + All-frequency lighting



1. High-Order Wavelets

2. Spherical Radial Basis Functions

# Why high-order wavelets?

- Representation should be tailored to the signal
- Smooth high-order wavelets (e.g. Daubechies-4) require an order of magnitude less coefficients to represent a smooth signal



environment lighting



Haar

Daubechies

16

32

64

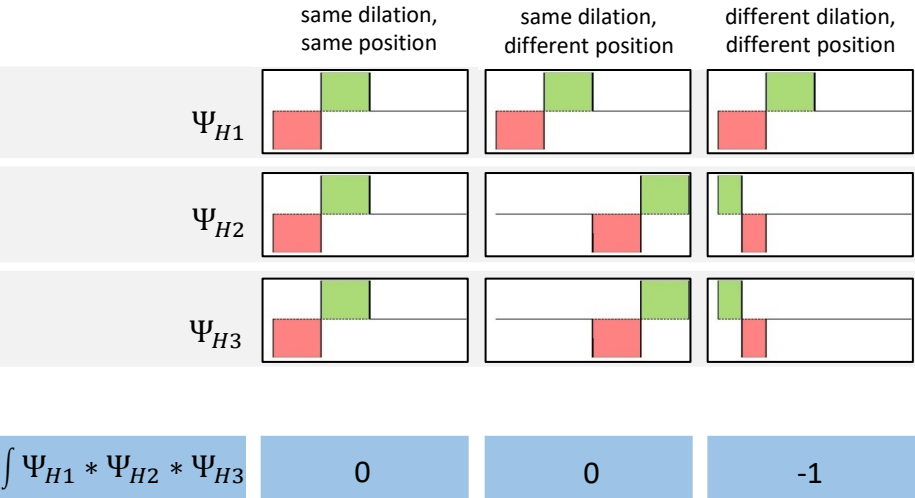
128

256

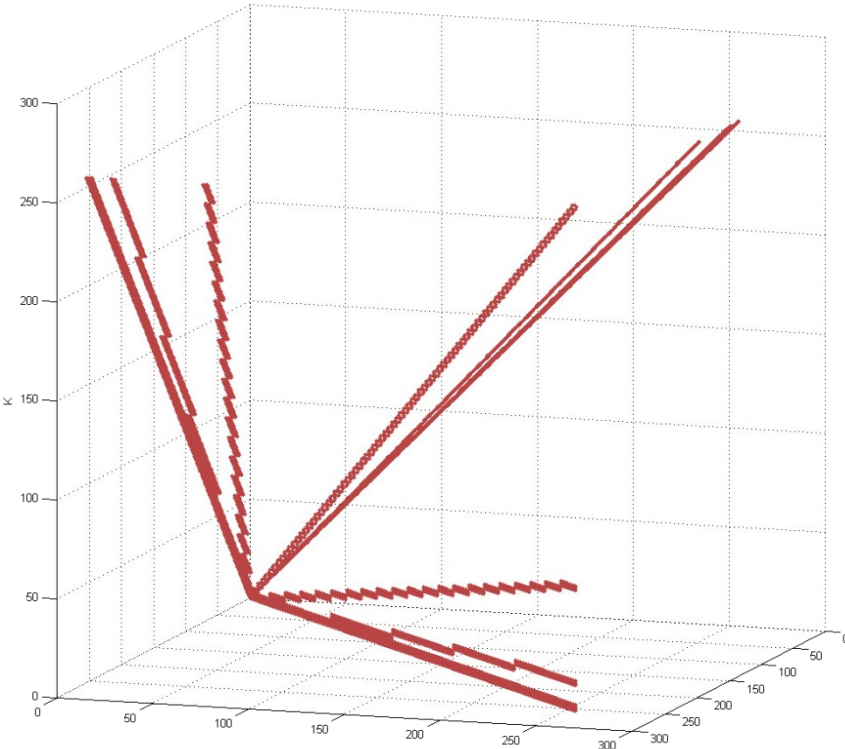
increasing number of coefficients

# Why is it so difficult to use high-order wavelets?

$$C_{ijk} = \int_{\Omega} \Psi_i(\omega)\Psi_j(\omega)\Psi_k(\omega)d\omega$$



Haar wavelets

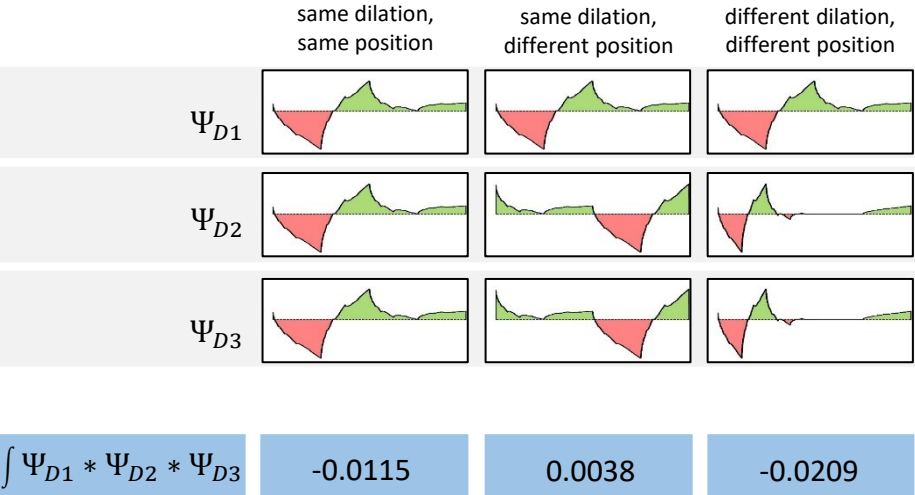


tensor of binding coefficients

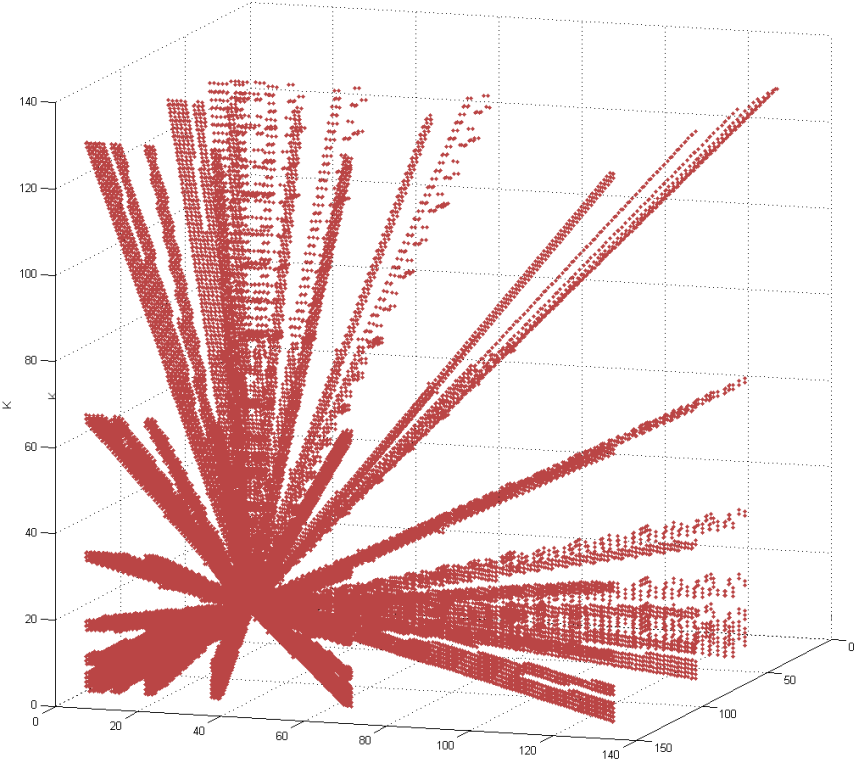


# Why is it so difficult to use high-order wavelets?

$$C_{ijk} = \int_{\Omega} \Psi_i(\omega)\Psi_j(\omega)\Psi_k(\omega)d\omega$$



Daubechies wavelets



tensor of binding coefficients

# General Wavelet Tripling Coefficient Theorem

---

- Naive approach
- Hierarchical approach
- Symmetry
- Wavelet sliding
- Vanishing moments

## *Test example*

- $D$ : dimensionality of the integral (double, triple, quadruple)
- $S_i$ : signal of  $r \times r$  resolution ( $i = 1, \dots, D$ )
- $N$ : number of dilations and translations of the basis function for  $S_i$

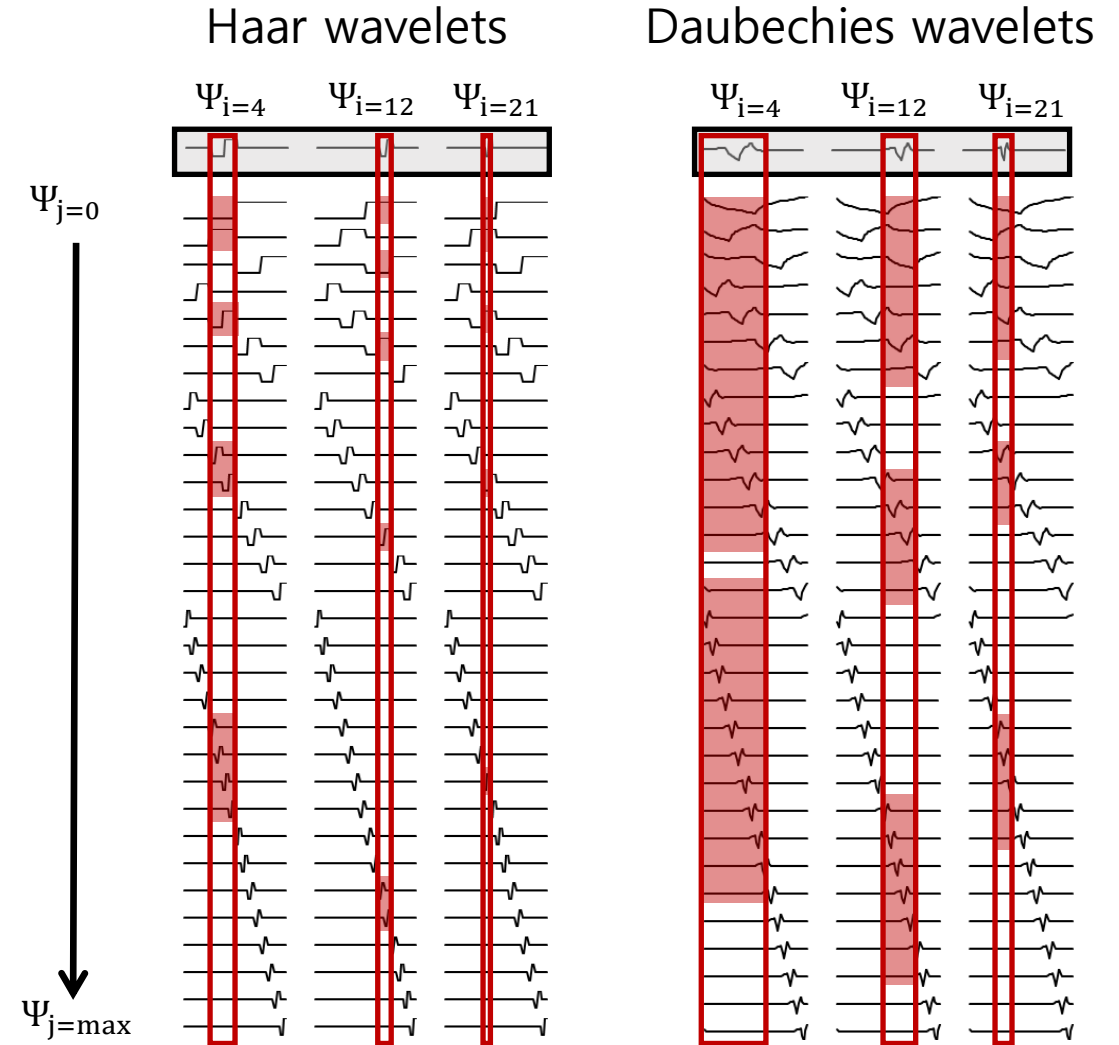
# General Wavelet Tripling Coefficient Theorem

---

- Naive approach
  - Iterate over all binding coefficients
  - $O(N^D r^2)$ 
    - **$4,7 \times 10^{21}$  operations**
    - $r = 512, N = 262144, D = 3$

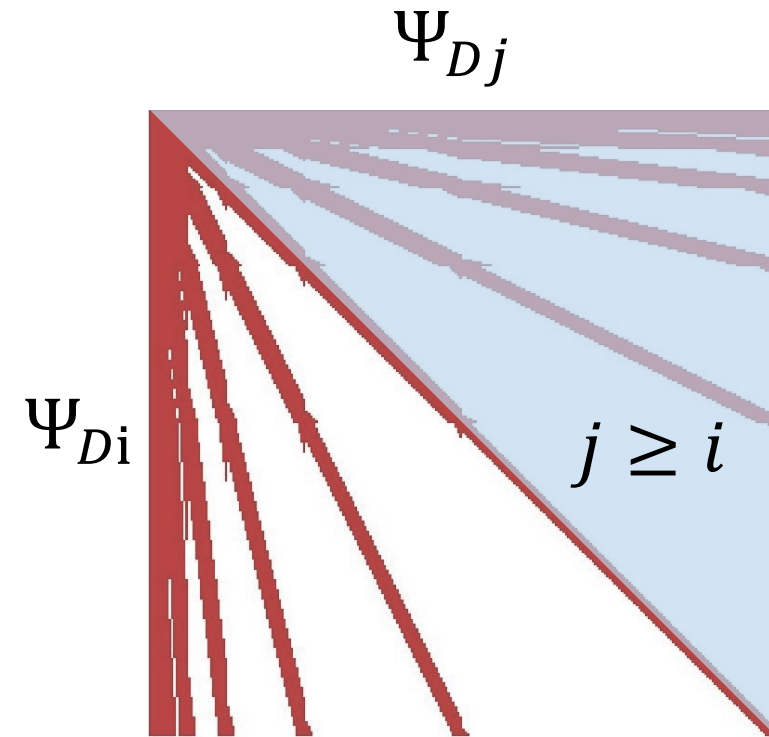
# General Wavelet Tripling Coefficient Theorem

- Hierarchical approach
  - Exploiting **support** of wavelet
  - Non-overlapping cases can be **skipped**
- $O(NC (\log N)^{D-1} r^2)$ 
  - $C$  relates to the enlargement of support
  - **$3,2 \times 10^{13}$  operations**
  - $r = 512, N = 262144, D = 3$



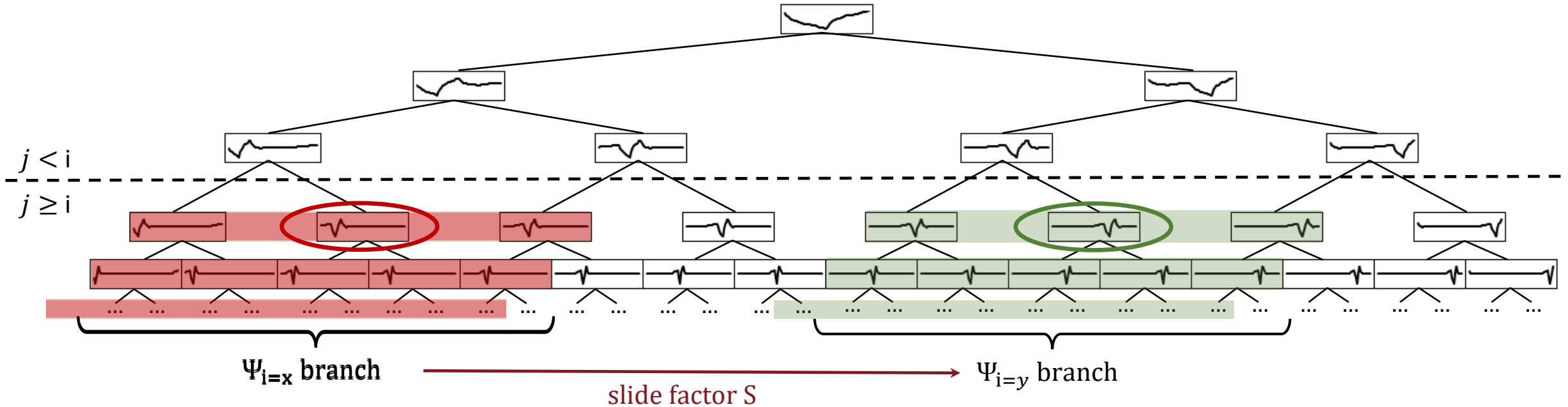
# General Wavelet Tripling Coefficient Theorem

- Symmetry
  - Homogeneous products
  - $\int \Psi_i \Psi_j = \int \Psi_j \Psi_i$



# General Wavelet Tripling Coefficient Theorem

- Wavelet sliding
  - Reuse duplicate branches in the tensor



$$\text{slide factor } S = (y - x) \times \text{support}(\Psi_{i=x})$$

# General Wavelet Tripling Coefficient Theorem

---

- Wavelet sliding
  - Reuse **duplicate branches** in the tensor
  - Calculate products for  **$\log N$  branches** instead of  $N$  branches
- $O(C (\log N)^D r^2)$ 
  - $C$  relates to the enlargement of support
  - **$1,5 \times 10^9$  operations**
  - $r = 512, N = 262144, D = 3$

# General Wavelet Tripling Coefficient Theorem

---

- Vanishing moments
  - High-order wavelets provide **more vanishing moments**
  - Causes **zero integrals** for certain translations within their support
  - **Increased sparsity** in tensor of binding coefficients
  - Identified and incorporated in wavelet sliding algorithm



# Sparseness – $8 \times 8$

$\Psi_i$	$\Psi_j$	$\Psi_k$	resolution	non-zero coeffs	total coeffs	sparseness
Haar-2	Haar-2	Haar-2	$8 \times 8$	1288	2.6E+5	<b>0.4913%</b>
Daub-4	Daub-4	Daub-4	$8 \times 8$	99088	2.6E+5	<b>37.7991%</b>
Daub-6	Daub-6	Daub-6	$8 \times 8$	214252	2.6E+5	<b>81.7307%</b>
Coiflet-5	Coiflet-5	Coiflet-5	$8 \times 8$	186706	2.6E+5	<b>71.2227%</b>
Haar-2	Daub-4	Daub-4	$8 \times 8$	31960	2.6E+5	<b>12.1918%</b>
Haar-2	Coiflet-5	Coiflet-5	$8 \times 8$	59902	2.6E+5	<b>22.8508%</b>

# Sparseness - $16 \times 16$

$\Psi_i$	$\Psi_j$	$\Psi_k$	resolution	non-zero coeffs	total coeffs	sparseness
Haar-2	Haar-2	Haar-2	$16 \times 16$	1288	1.6E+7	<b>0.0443%</b>
Daub-4	Daub-4	Daub-4	$16 \times 16$	99088	1.6E+7	<b>8.2263%</b>
Daub-6	Daub-6	Daub-6	$16 \times 16$	214252	1.6E+7	<b>25.9928%</b>
Coiflet-5	Coiflet-5	Coiflet-5	$16 \times 16$	186706	1.6E+7	<b>20.8433%</b>
Haar-2	Daub-4	Daub-4	$16 \times 16$	31960	1.6E+7	<b>1.6133%</b>
Haar-2	Coiflet-5	Coiflet-5	$16 \times 16$	59902	1.6E+7	<b>3.7315%</b>

# Sparseness - $32 \times 32$

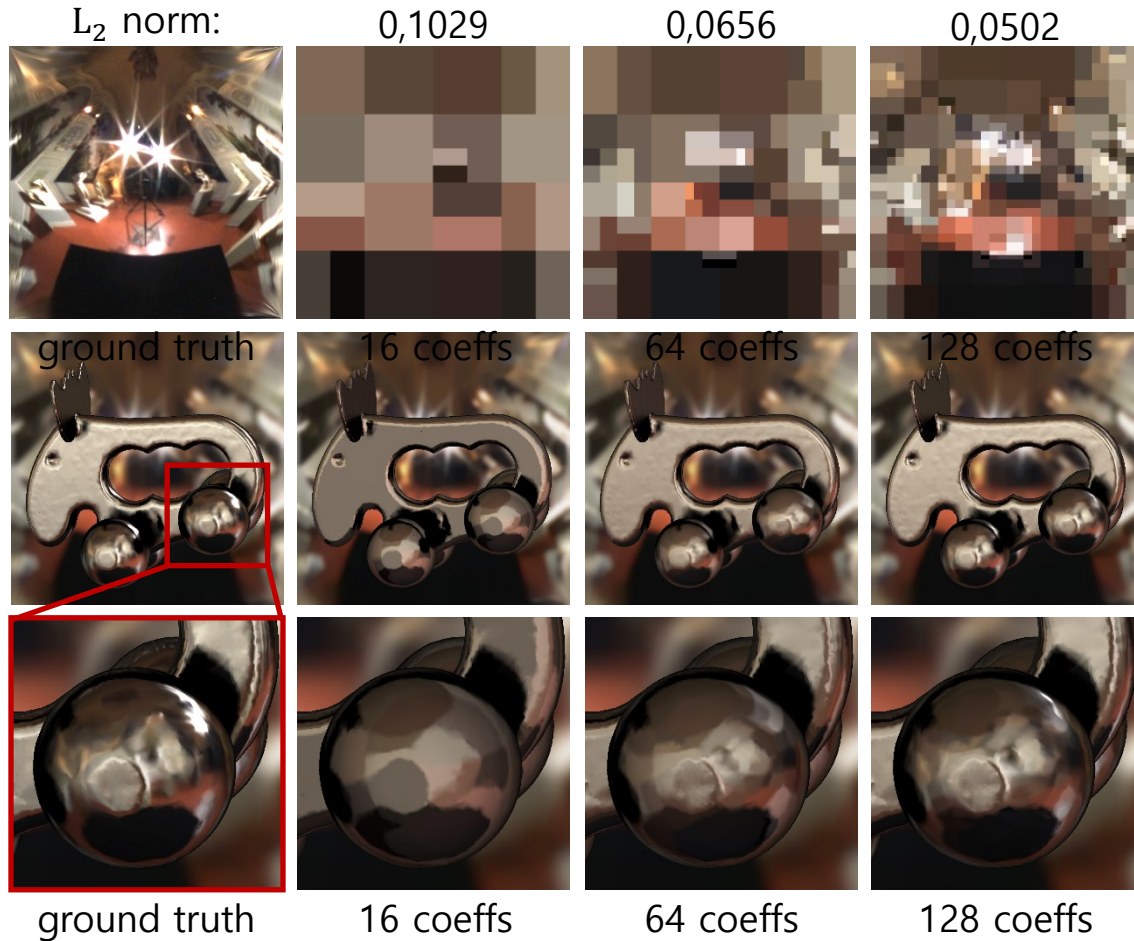
$\Psi_i$	$\Psi_j$	$\Psi_k$	resolution	non-zero coeffs	total coeffs	sparseness
Haar-2	Haar-2	Haar-2	$32 \times 32$	38920	1.1E+9	<b>0.0036%</b>
Daub-4	Daub-4	Daub-4	$32 \times 32$	9720040	1.1E+9	<b>0.9052%</b>
Daub-6	Daub-6	Daub-6	$32 \times 32$	29582032	1.1E+9	<b>2.7550%</b>
Coiflet-5	Coiflet-5	Coiflet-5	$32 \times 32$	16408816	1.1E+9	<b>1.5282%</b>
Haar-2	Daub-4	Daub-4	$32 \times 32$	1555120	1.1E+9	<b>0.1448%</b>
Haar-2	Coiflet-5	Coiflet-5	$32 \times 32$	2715112	1.1E+9	<b>0.2529%</b>

# Sparseness - $64 \times 64$

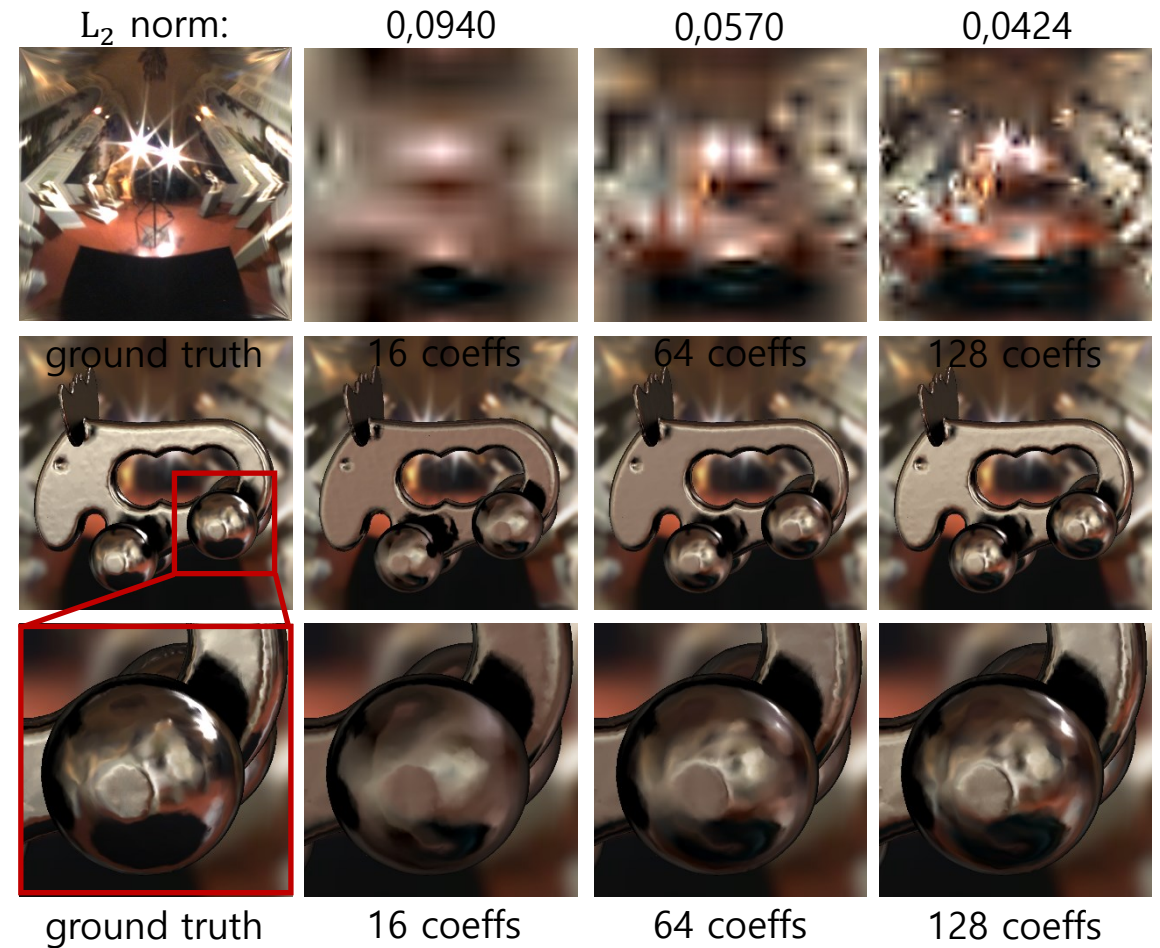
$\Psi_i$	$\Psi_j$	$\Psi_k$	resolution	non-zero coeffs	total coeffs	sparseness
Haar-2	Haar-2	Haar-2	$64 \times 64$	192520	6.9E+10	<b>0.0003%</b>
Daub-4	Daub-4	Daub-4	$64 \times 64$	47918464	6.9E+10	<b>0.0697%</b>
Daub-6	Daub-6	Daub-6	$64 \times 64$	145473456	6.9E+10	<b>0.2117%</b>
Coiflet-5	Coiflet-5	Coiflet-5	$64 \times 64$	48918464	6.9E+10	<b>0.0712%</b>
Haar-2	Daub-4	Daub-4	$64 \times 64$	7327168	6.9E+10	<b>0.0107%</b>
Haar-2	Coiflet-5	Coiflet-5	$64 \times 64$	8699044	6.9E+10	<b>0.0127%</b>

# Render Application

## Haar wavelets



## Daubechies-6 wavelets



# What Is Wrong with Wavelets?

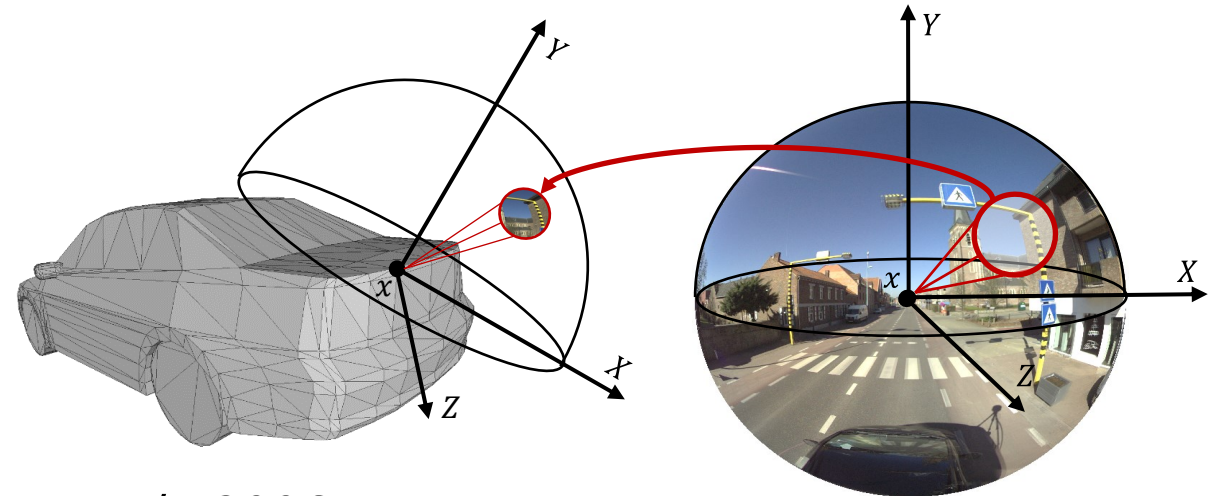
- 2D Haar wavelets [Ng et al, 2004]
- Spherical Haar wavelets [Put et al., 2014]
- High-order wavelets [Michiels et al., 2014]

+ Few coefficients

+ All-frequency

- Preprocessing

- No efficient rotation operator [Wang et al., 2006]



1. High-Order Wavelets

**2. Spherical Radial Basis Functions**

# Spherical Radial Basis Functions (SRBFs)

- Radial Basis Functions, defined on the sphere

- Poisson( $\omega, \mathbf{c}, \lambda, \mu$ ) =  $\mu \frac{1-\lambda^2}{(1-2\lambda(\omega \cdot \mathbf{c})+\lambda^2)^{3/2}}$
- Multiquadratic( $\omega, \mathbf{c}, \lambda, \mu$ ) =  $\mu \sqrt{1 + (\lambda(\omega \cdot \mathbf{c}))^2}$
- **Gaussian**( $\omega, \mathbf{c}, \lambda, \mu$ ) =  $\mu e^{\lambda(\omega \cdot \mathbf{c}-1)}$

+ All-frequency

+ Decent compression performance

+ Efficient rotation operator

+ Analytic evaluation of the binding coefficients



# Triple product rendering with SRBFs

	all-frequency	dynamic lighting	dynamic geometry	dynamic brdf
Sloan et al., 2003	×	×	×	×
Ng et al., 2004	✓	×	×	×
Tsai and Shih, 2006	✓	rotation only	×	×
Haber et al., 2009	✓	×	×	×
Wang et al., 2009	✓	rotation only	×	✓
Lam et al., 2010	✓	rotation only	×	×
Iwasaki et al., 2012	✓	rotation only	low-poly	✓
our method	✓	✓	✓	✓

- Current techniques constrain one or several factors
- + Our approach is able to **dynamically construct and update all three factors**

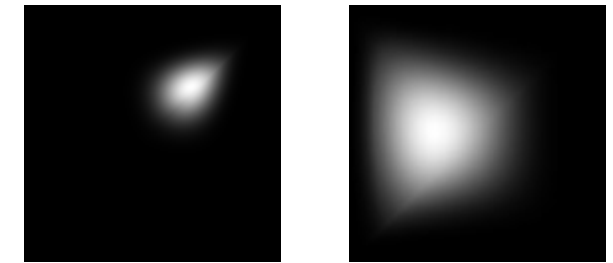
## SRBFs

- + Directly approximated with Gaussian lobes
- + Phong, Cook-Torrance, Ward, Blinn-Phong, ...

*[Wang et al., 2009]*

## Previous approaches

- Sampled in pixel domain for each BRDF slice ( $x, \omega_i, \omega_o$ )
- On-the-fly transformation to wavelets



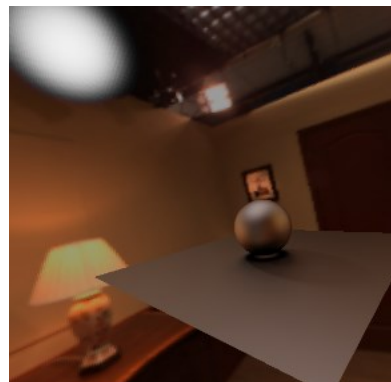
*[Haber et al., 2009]*



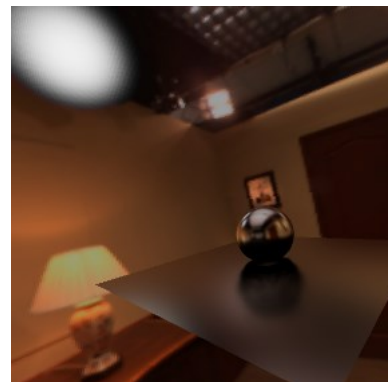
*Lambertian*



*Diffuse Phong*



*Glossy Phong*



*Specular Phong*

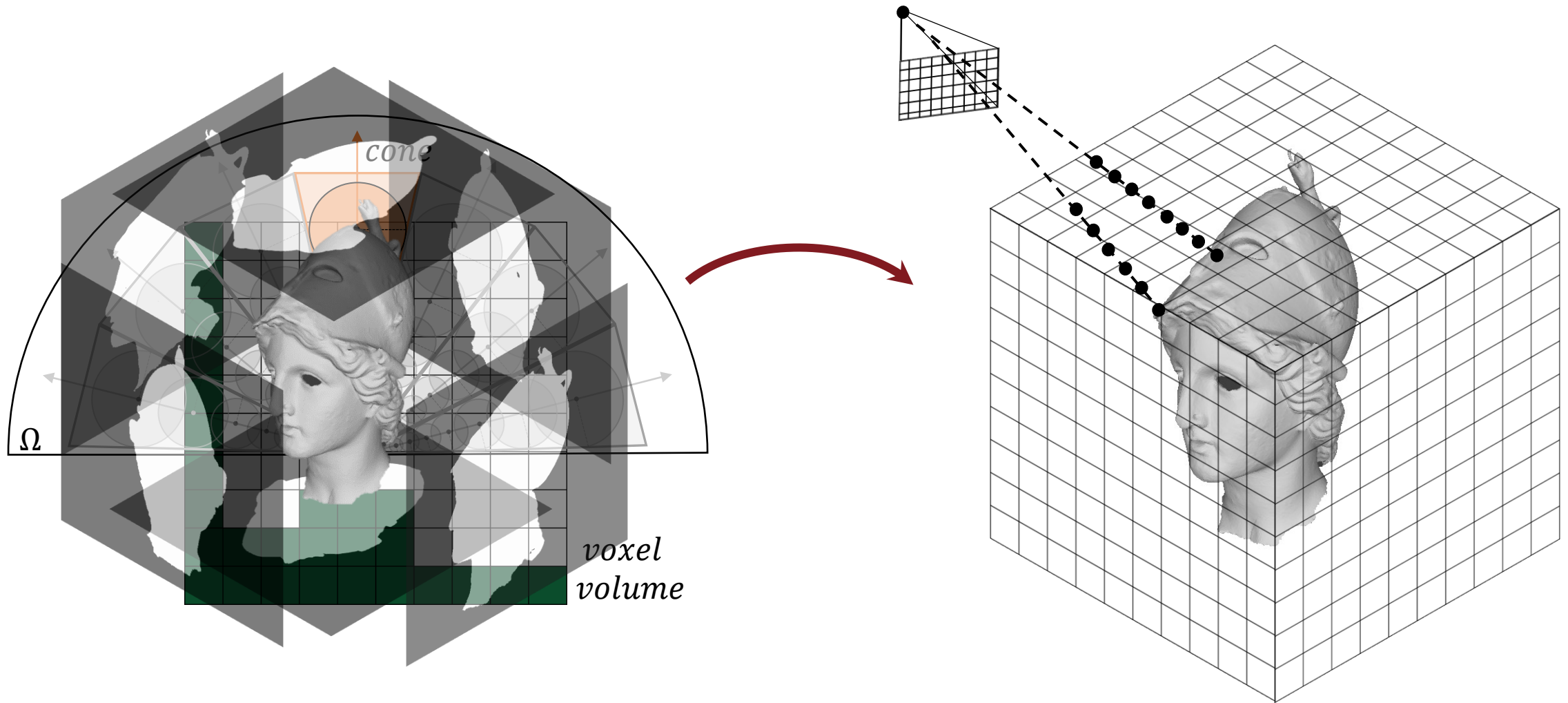
## Our approach (SRBFs)

- + Combination of PRT and voxelization
- + **One-pass voxelization** [Crassin and Green, 2012]
- + Mapping of **visibility SRBFs** to voxel cones
- + Entirely on **GPU**

## Previous approaches

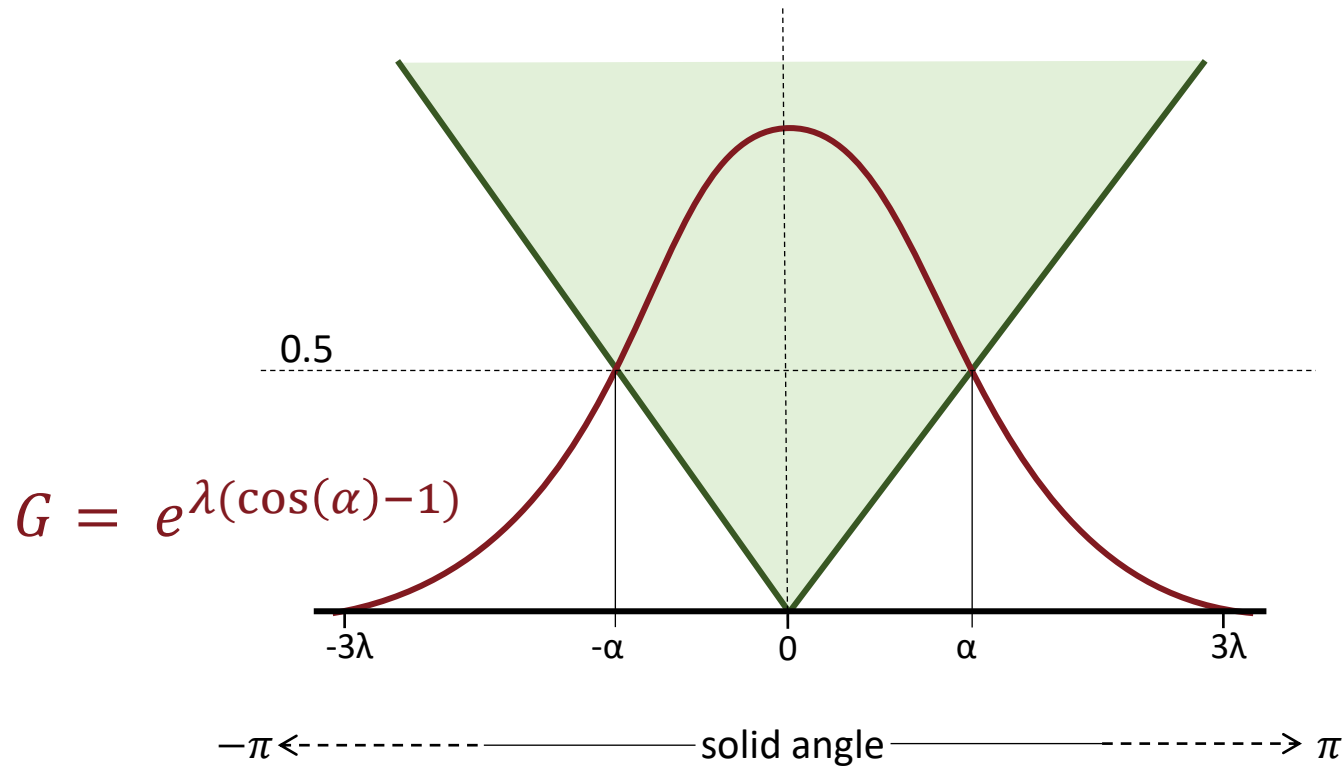
- Rely on precomputation
- Limited to static scenes
- Ray tracing in pixel domain  
*[Haber et al., 2009]*
- Approximated with Spherical Signed Distance Functions  
*[Wang et al., 2009]*
- Projecting bounding volumes on hemisphere  
*[Iwasaki et al., 2012]*

# Dynamic Visibility – Cone tracing



# Dynamic Visibility – Mapping SRBF to Visibility Cone

- Sampling of visibility in the SRBF lobe
- Mapping SRBF to a corresponding visibility cone

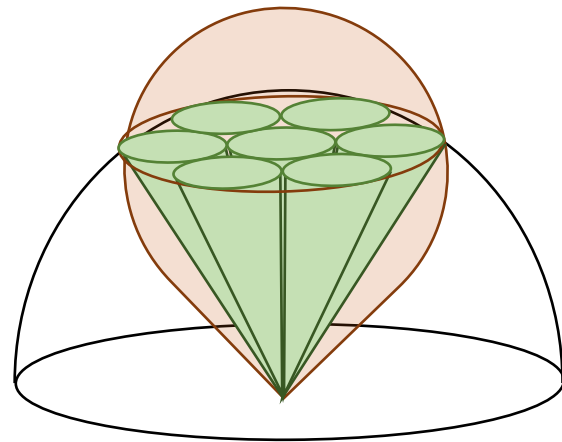


$$G = e^{\lambda(\cos(\alpha)-1)} = 0.5$$

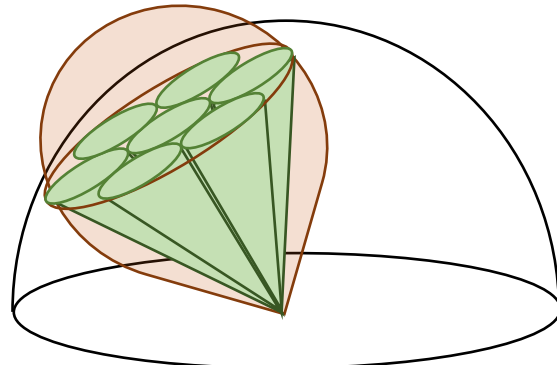
$$\Leftrightarrow \alpha = \cos^{-1} \left( \frac{\log(0.5)}{\lambda} + 1 \right)$$

# Dynamic Visibility – Subsampling Scheme

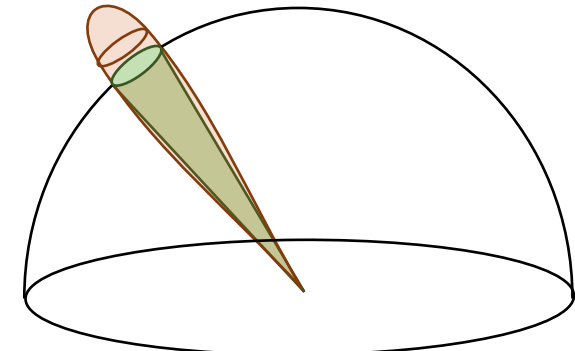
- **Subsampling** of SRBF lobe is essential
  - Avoid integration of high-frequency **visibility detail** over larger area of the hemisphere
  - **Circle packing**: maximize density of subsampled cones
  - Adaptive subsampling based on **BRDF lobe**



*Lambertian*



*Glossy BRDF*

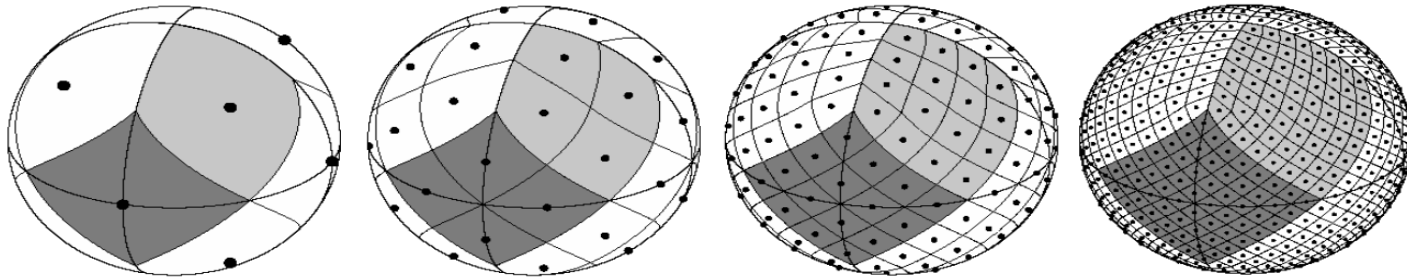


*Specular BRDF*

# Dynamic Lighting

## Our approach (SRBFs)

- + HDR omnidirectional photo/video
- + SRBF fitting
- + Multi-scale algorithm using fixed grid
- + SRBF centers defined by Healpix distribution
- + Entirely on GPU

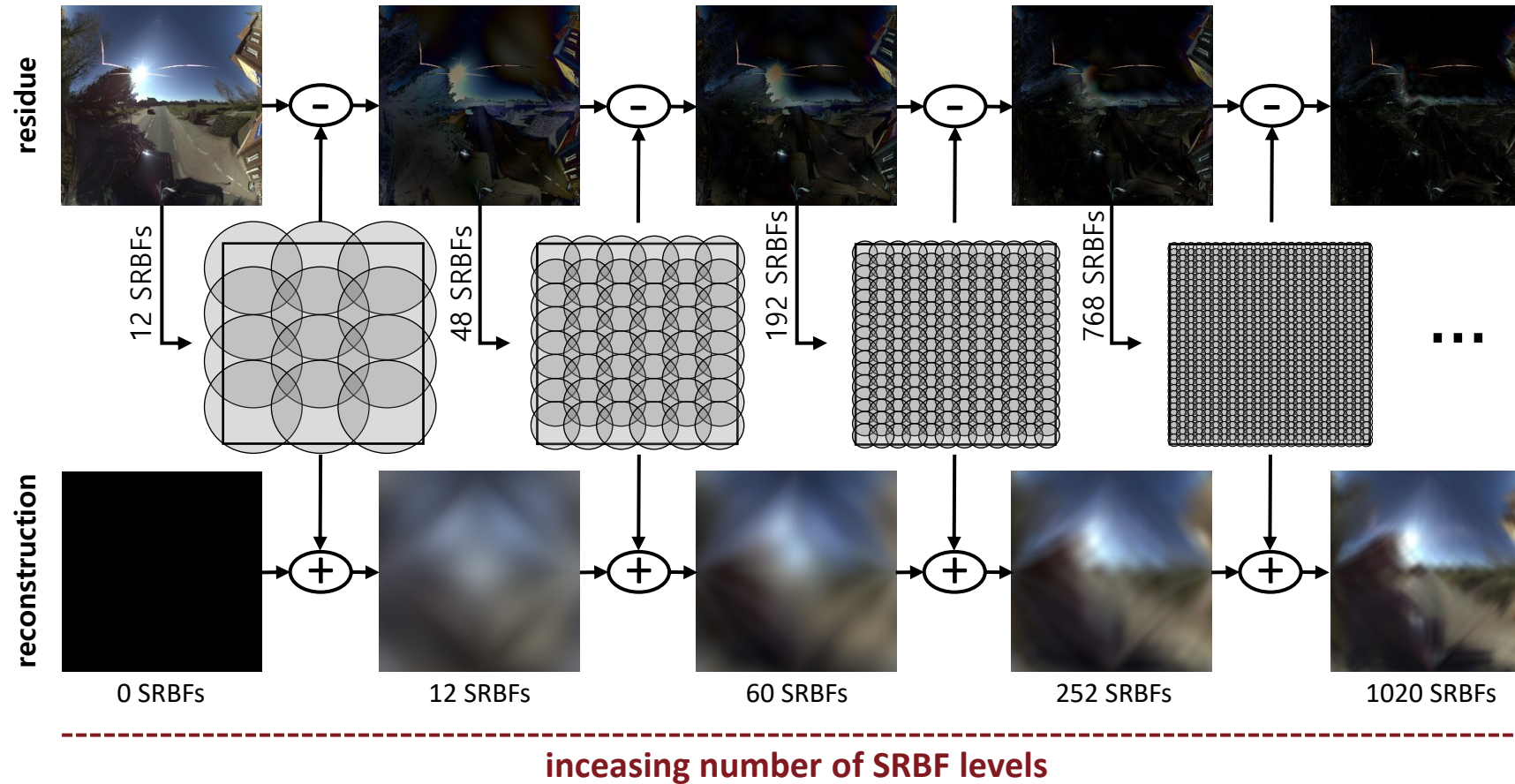


## Previous approaches

- Optimization [Tsai and Shih, 2006]
  - + Good compression
  - Slow
- Least-square projection [Lam et al., 2010]

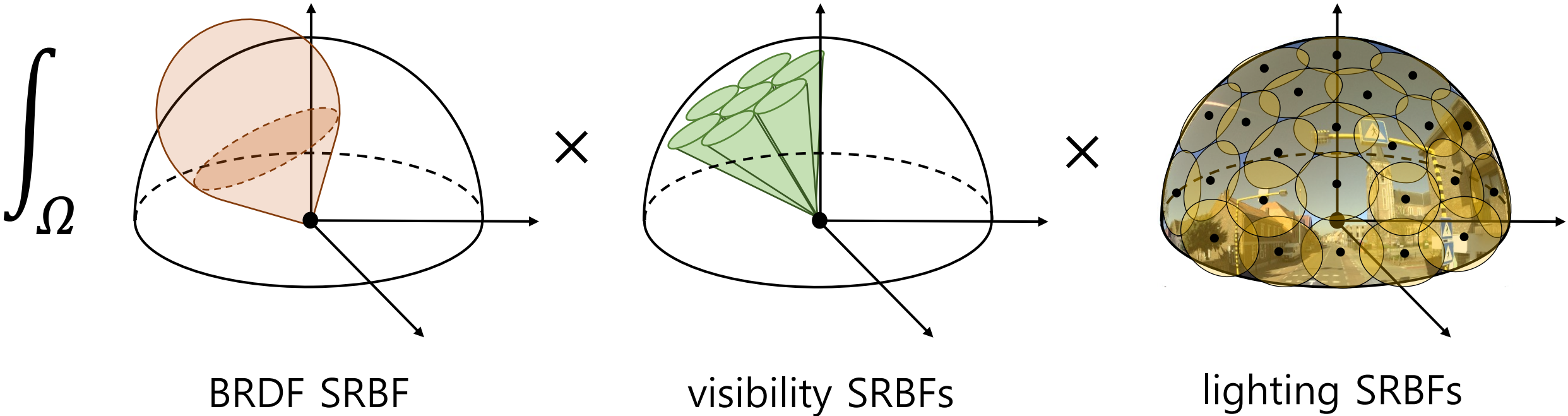
# Dynamic Lighting

- Multi-scale residual transform

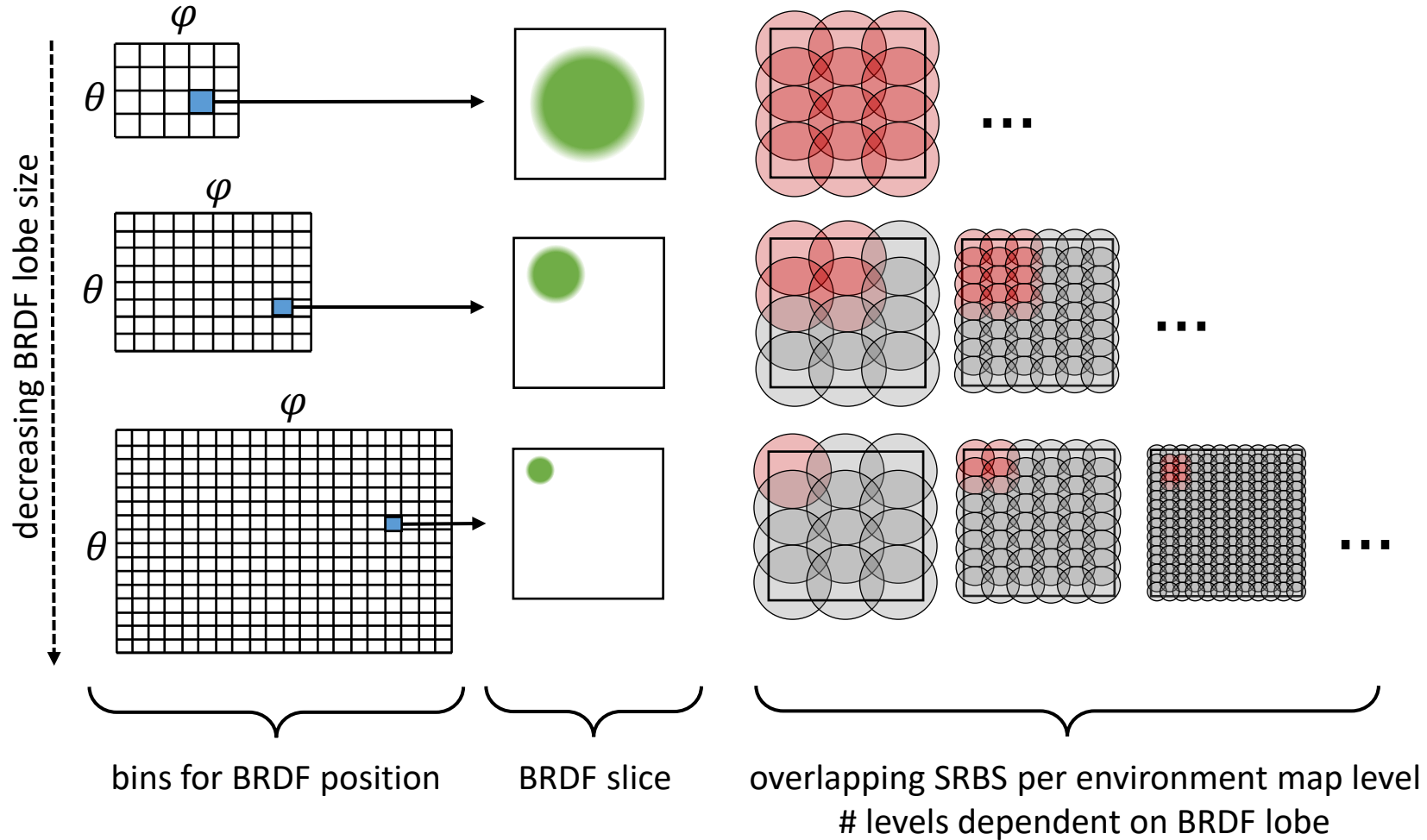




# Overlapping Lighting SRBFs



# Overlapping Lighting SRBFs

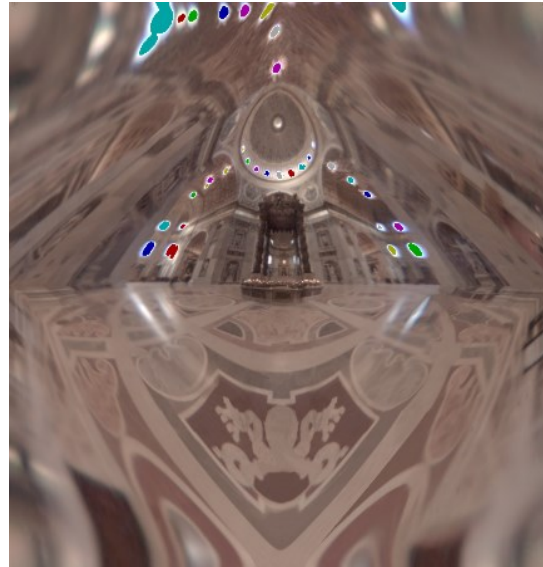


# Peak Detection

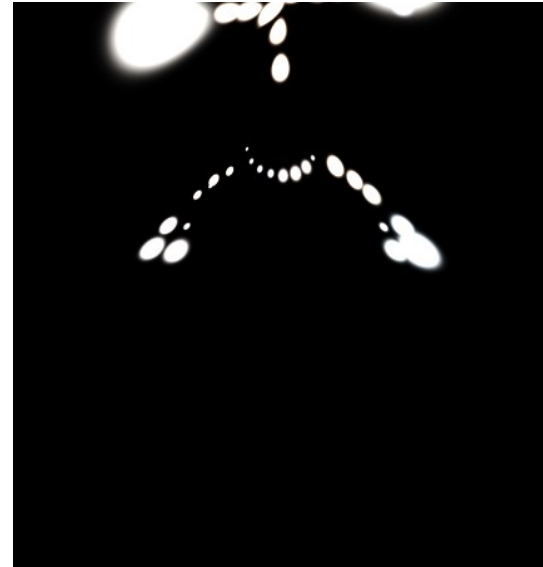
- Problem: bright area light sources
  - Requires too fine-grained subsampling of visibility cones
- + Solution: **peak detection**
  - Treated as a special case
  - Thresholding / connected components / fitting



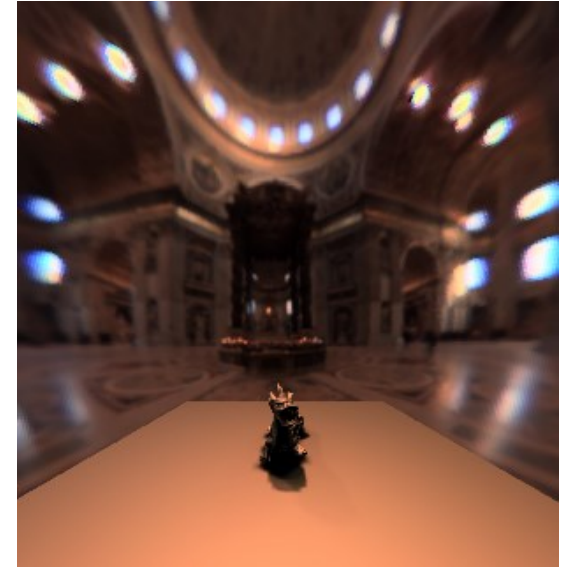
environment map



connected components

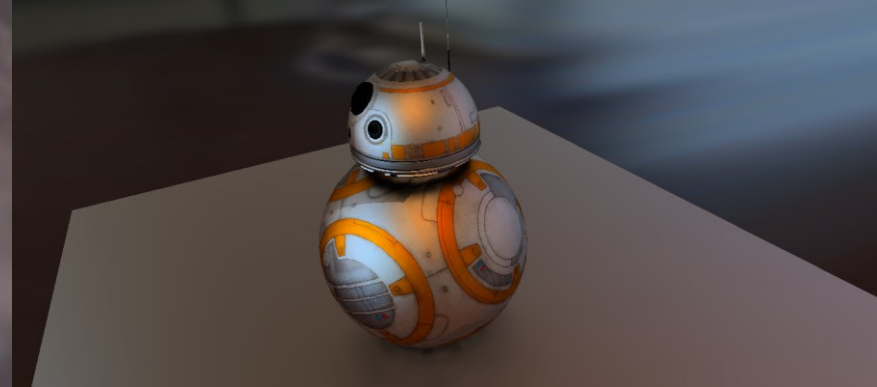


SRBF fitting

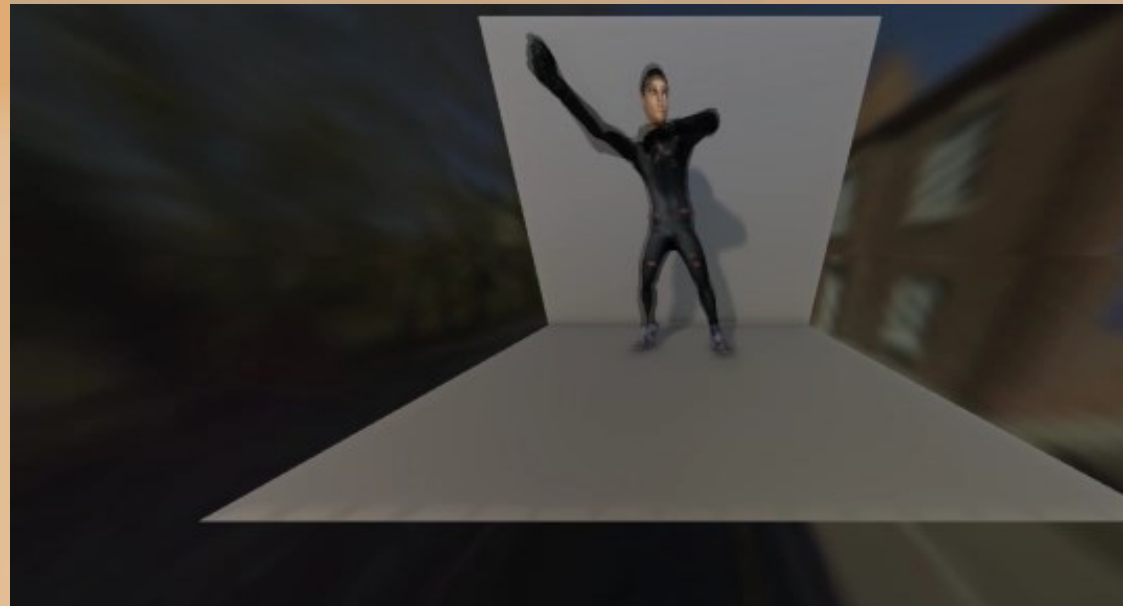


rendering

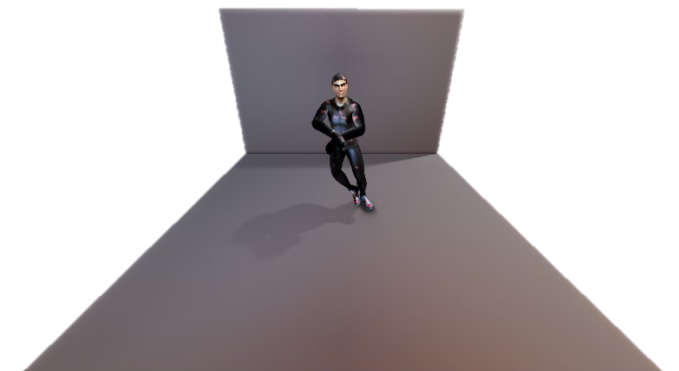
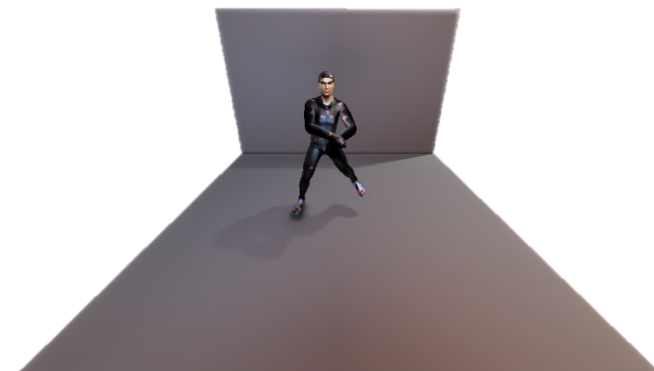
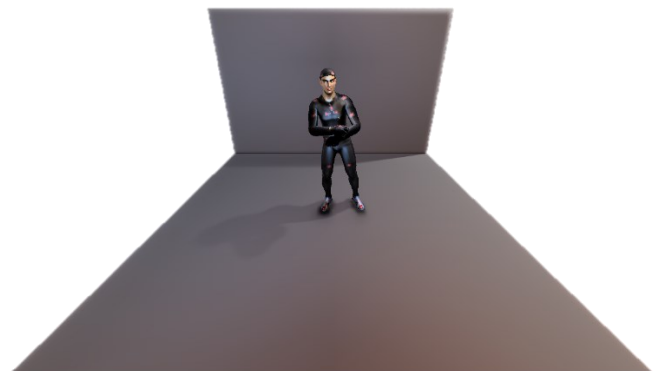
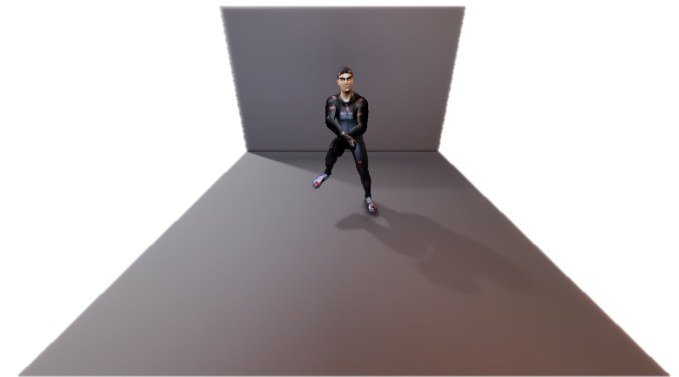
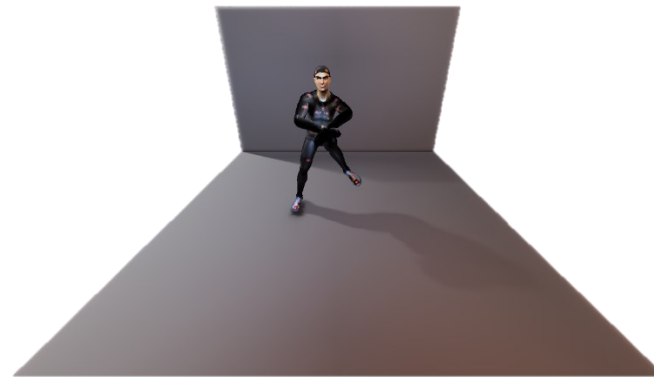
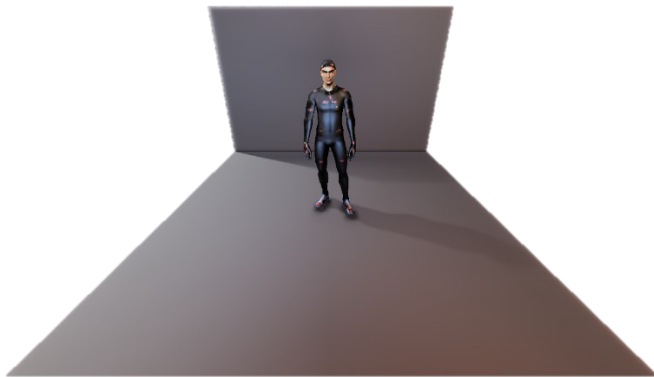
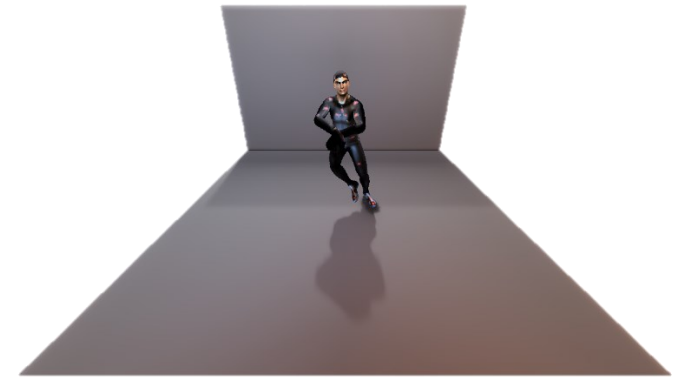
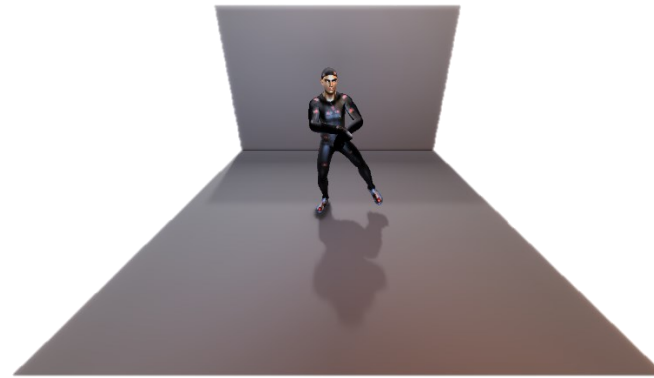
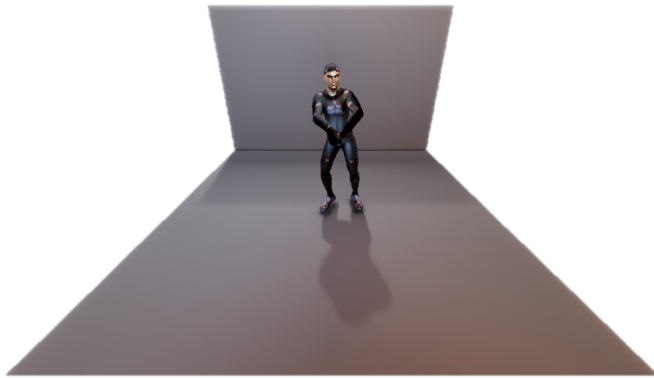
# Results



# Results



# Results



## 1. Relighting of **Virtual Objects**



## 2. Relighting of **Real Objects**



# Relighting of Virtual Objects





# Relighting of **Virtual Objects**

## 1. Relighting of **Virtual Objects**



## 2. Relighting of **Real Objects**



# Relighting of Real Objects

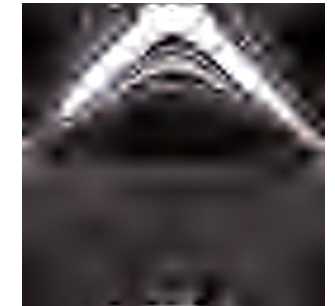
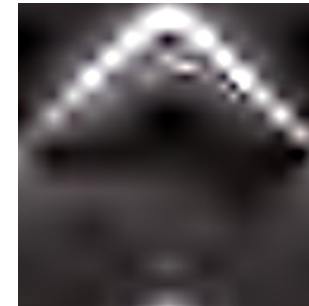
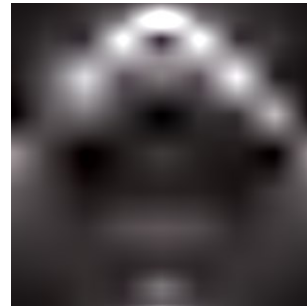
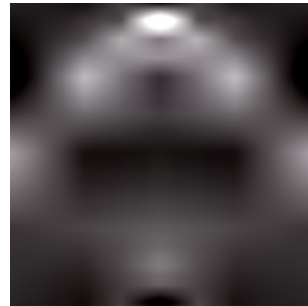
- Inverse rendering using wavelets [Haber et al., 2009]
  - Hierarchical refinement using smooth high-order wavelets



environment map



input image



# Relighting of Real Objects

- Inverse rendering using wavelets [*Haber et al., 2009*]
  - Hierarchical refinement using high-order wavelets
  - Temporal information



input  
sequence



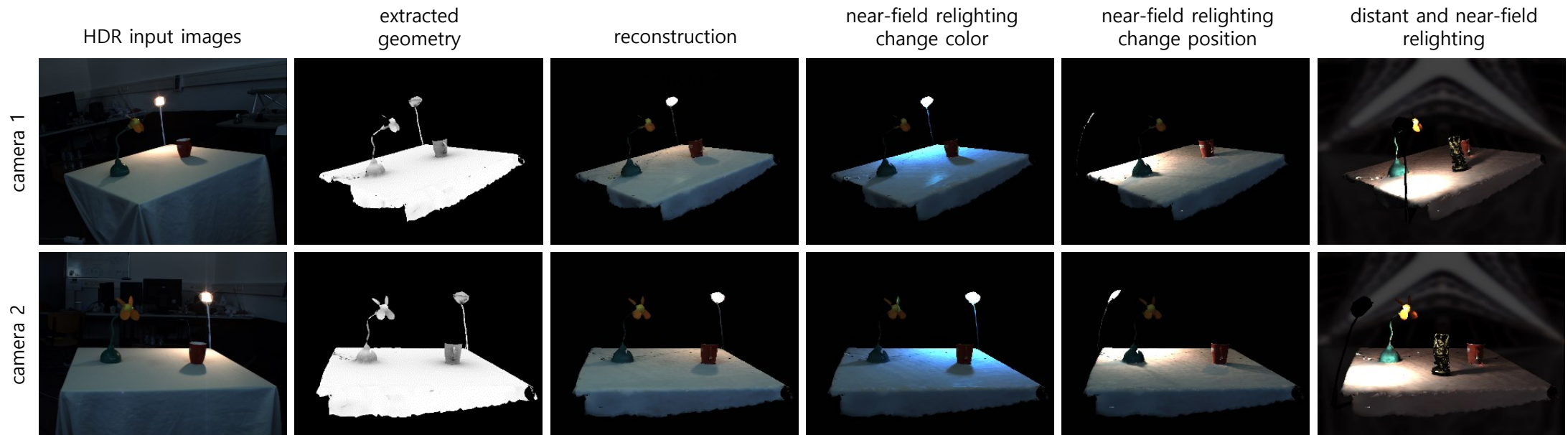
reconstructed  
sequence



relit with Uffizi  
environment map

# Relighting of Real Objects

- Inverse rendering using wavelets [Haber et al., 2009]
  - Hierarchical refinement using high-order wavelets
  - Temporal information
  - Near-field lighting

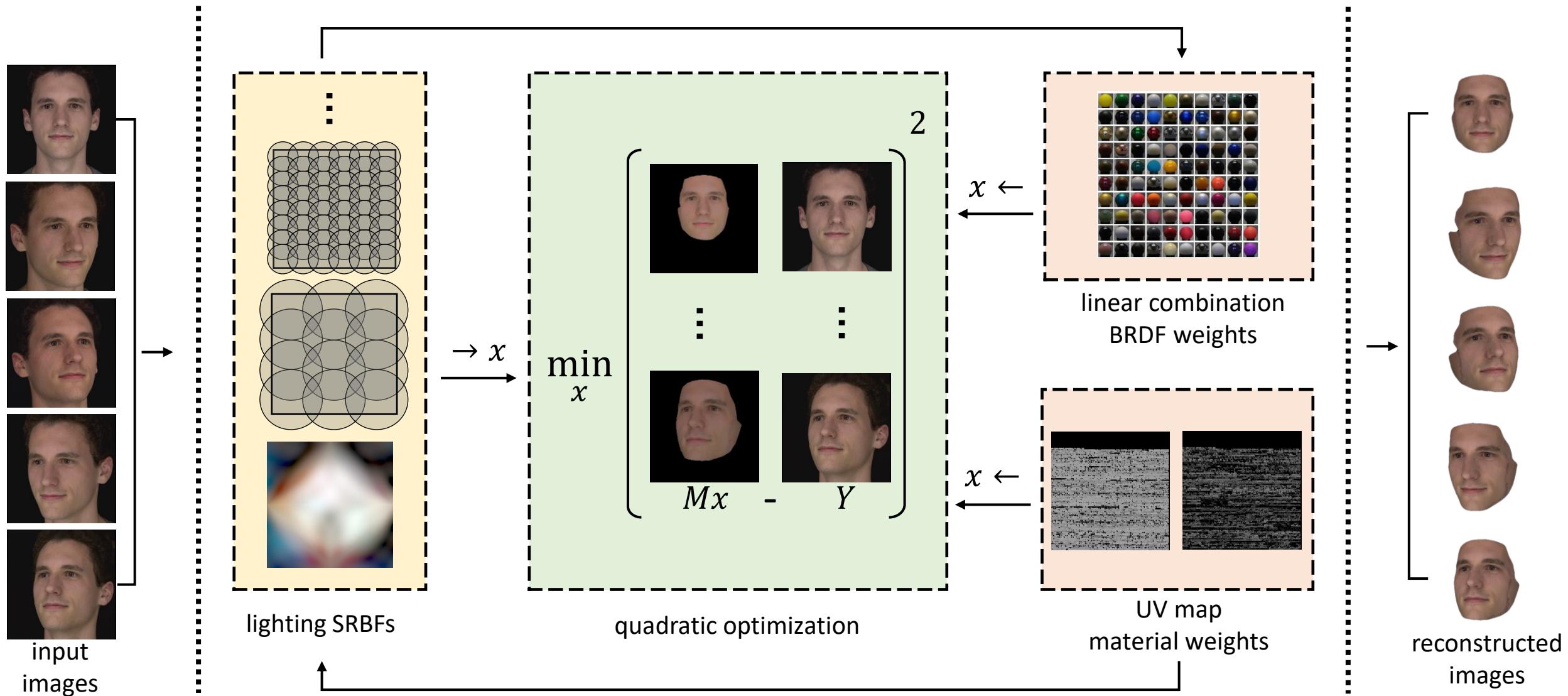


# Relighting of Real Objects

---

- Inverse rendering using wavelets [*Haber et al., 2009*]
  - Hierarchical refinement using high-order wavelets
  - Temporal information
  - Near-field lighting
- Inverse rendering using SRBFs [*Haber et al., 2009*]

# Relighting of Real Objects - Inverse Rendering using SRBFs



# Relighting of Real Objects - Inverse Rendering using SRBFs

	Haar wavelets [Haber et al., 2009]	SRBF [ours]
# lighting coefficients	1024	<b>1020</b>
$\Delta t$ optimize lighting	1089 s	<b>10 s</b>
# BRDF weights	2689 (per vertex)	<b>906240</b> (per texel)
$\Delta t$ optimize BRDF weights	3 s	<b>180 s</b>
$\Delta t$ optimize one BRDF weights	$9.4 \times 10^{-4}$ s	<b><math>1.9 \times 10^{-4}</math> s</b>
$\Delta t$ full optimization	184 min	<b>29 min</b>
$\Delta t$ rendering reconstruction	13.403 s	<b>&lt; 0.1 s</b>





# Conclusions

---

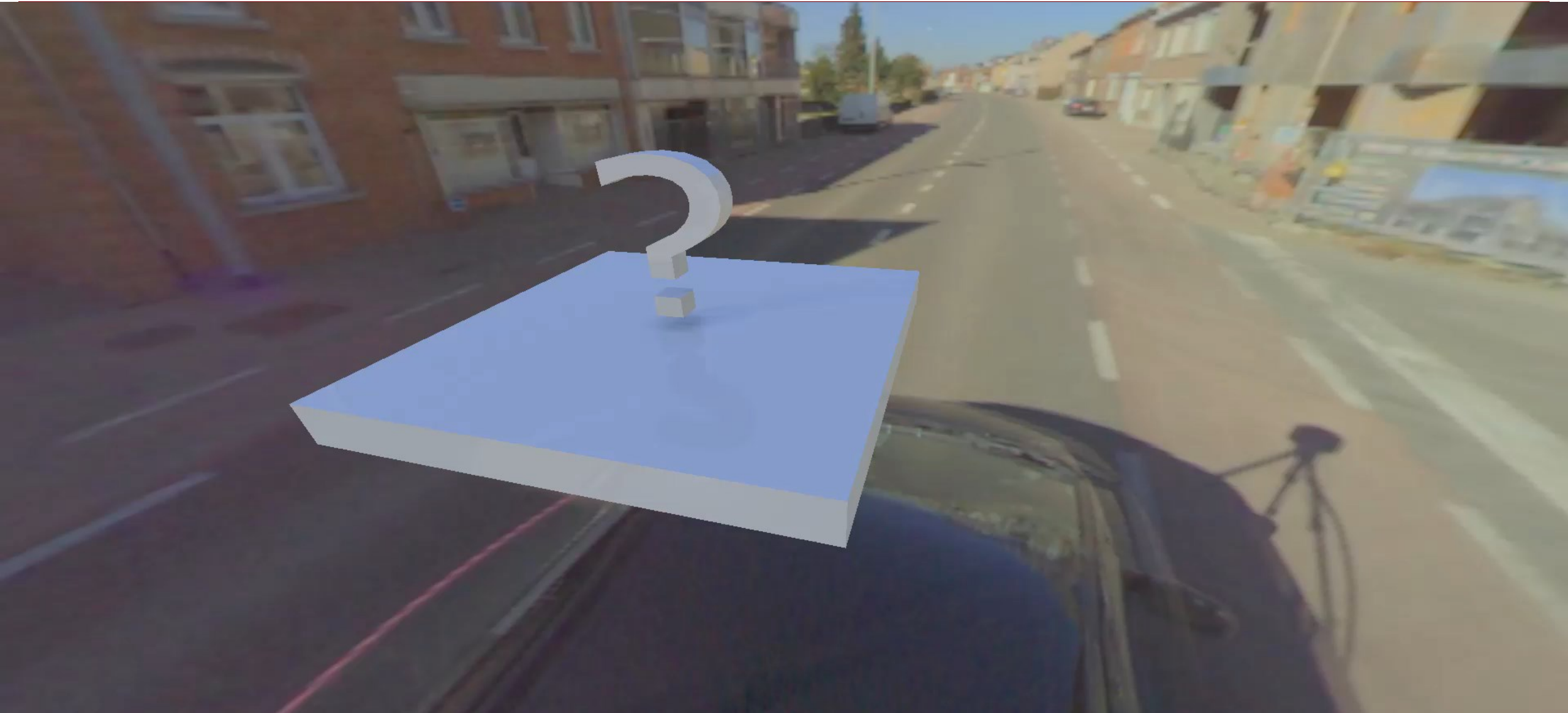
- General triple product theorem for high-order wavelets
  - + Triple product rendering for a **mixture of different wavelets bases**
  - + Wavelets **tailored** to the signal
  - + **Few coefficients** to estimate
  - Rather slow
- Dynamic triple product rendering using SRBFs
  - + **Interactive** and **real-time** triple product rendering
  - + All three **factors are dynamic**
  - + **Preview rendering** of estimates
  - Slightly more coefficients: quality/speed tradeoff

# Take Home Messages

---

- Underlying representation does have an impact on relighting applications
  - Representation tailored to the signal
  - Representation tailored to the application
  - Compression/time tradeoff

Thank you for your attention!



Thank you for your attention

---

# High-Order Wavelets in Render Application

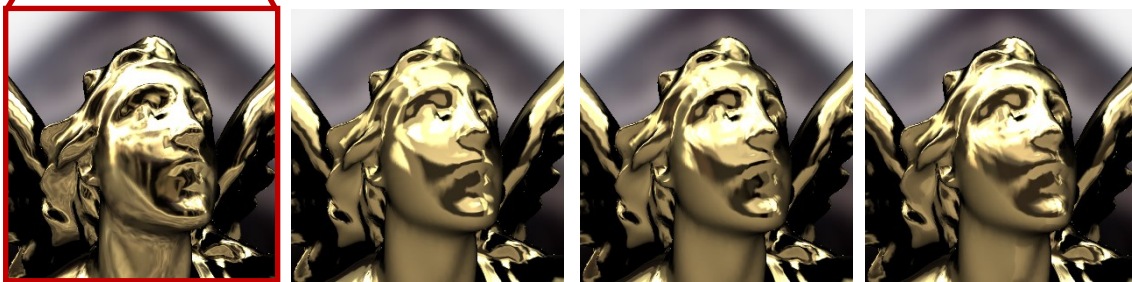
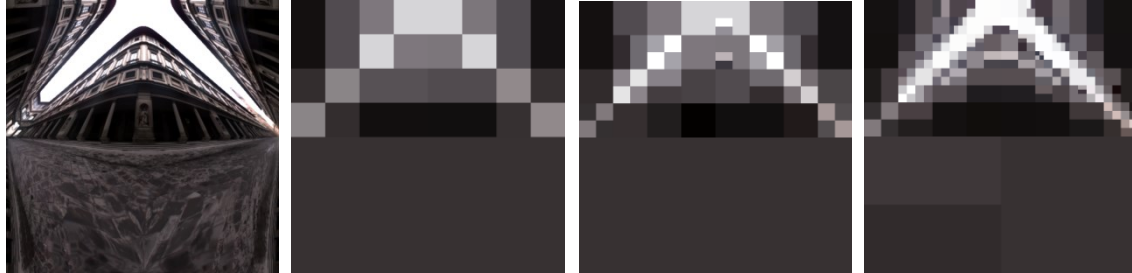
## Haar wavelets

$L_2$  norm:

0,0990

0,0803

0,0542



ground truth

16 coeffs

32 coeffs

128 coeffs

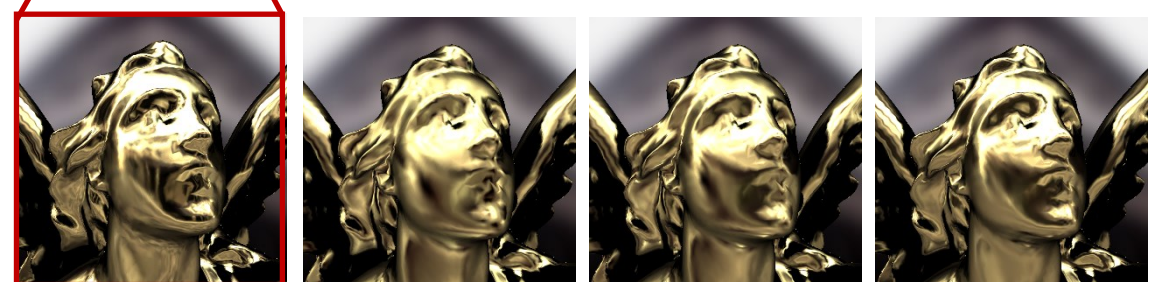
## Daubechies-6 wavelets

$L_2$  norm:

0,0921

0,0717

0,0475



ground truth

16 coeffs

32 coeffs

128 coeffs

# High-Order Wavelets in Render Application



# High-Order Wavelets in Render Application



optimal choice of wavelet basis



2D wavelet basis

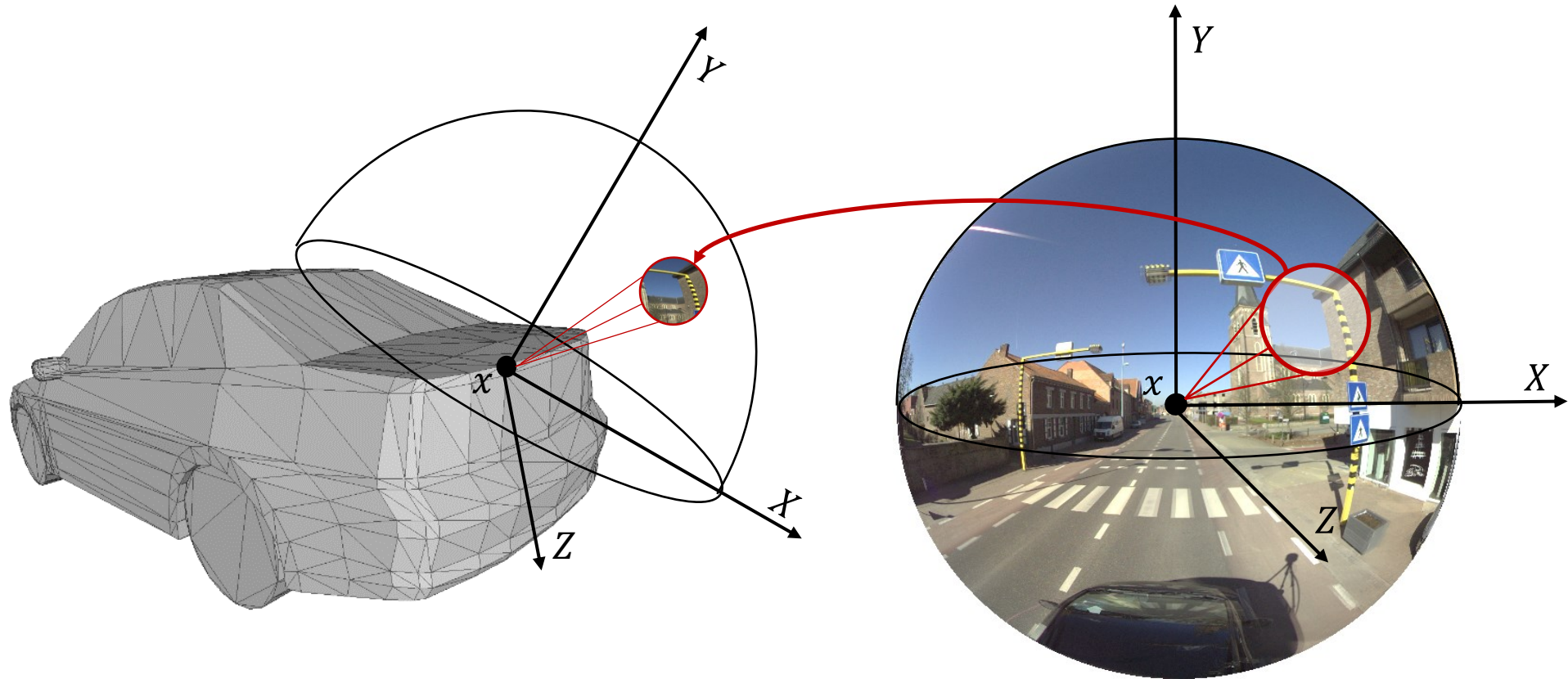
# Triple product rendering with SRBFs

## + Analytic Evaluation of the binding coefficients

$$\begin{aligned}C_{ijk} &= \int_{\Omega} \Psi_i(\omega) \Psi_j(\omega) \Psi_k(\omega) d\omega & \Psi_{i,j,k} &= G(\omega, \mathbf{c}, \lambda, \mu) = \mu e^{\lambda(\omega \cdot \mathbf{c} - 1)} \\&= \int_{\Omega} G(\omega, \mathbf{c}_V, \lambda_V, \mu_V) G(\omega, \mathbf{c}_L, \lambda_L, \mu_L) G(\omega, \mathbf{c}_\rho, \lambda_\rho, \mu_\rho) d\omega \\&= \mu_V \mu_L \mu_\rho e^{-(\lambda_V + \lambda_L + \lambda_\rho)} \int_{\Omega} e^{\omega \cdot (\lambda_V \mathbf{c}_V + \lambda_L \mathbf{c}_L + \lambda_\rho \mathbf{c}_\rho)} d\omega \\&= \mu_V \mu_L \mu_\rho e^{-(\lambda_V + \lambda_L + \lambda_\rho)} 4\pi \frac{\sinh \|\lambda_V \mathbf{c}_V + \lambda_L \mathbf{c}_L + \lambda_\rho \mathbf{c}_\rho\|}{\|\lambda_V \mathbf{c}_V + \lambda_L \mathbf{c}_L + \lambda_\rho \mathbf{c}_\rho\|} && [Tsai and Shih, 2006]\end{aligned}$$



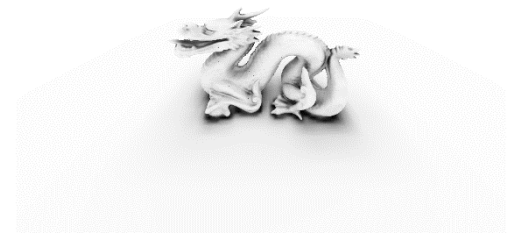
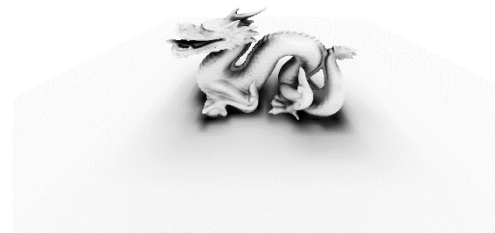
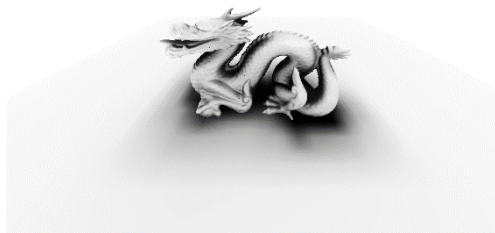
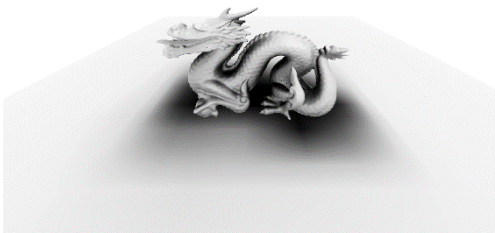
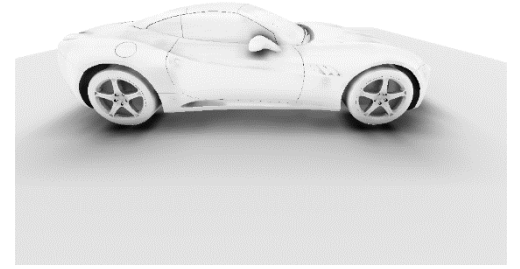
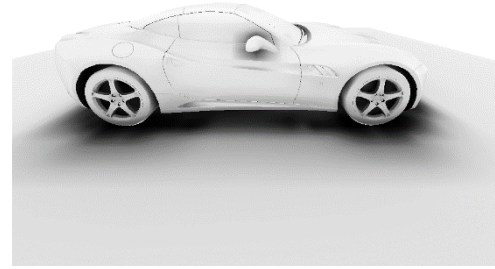
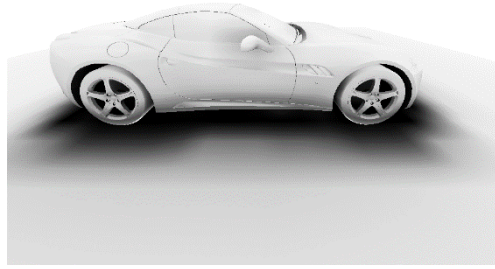
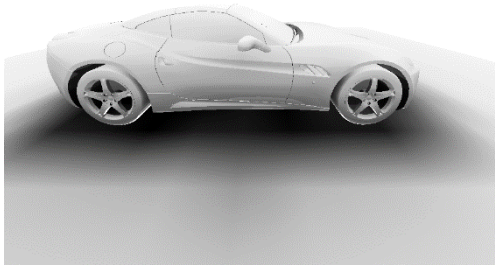
# Rotation Operator



# Overlapping Lighting SRBFs

bin level	BRDF $\lambda$	# bins	mean # overlapping SRBFs	max level
1	2,0	45	60	1
2	4,0	153	252	2
3	16,0	561	312	3
4	32,0	2145	608	4
5	128,0	2145	182	5
6	256,0	8320	109	5
7	512,0	33153	70	5

# Dynamic Visibility – Voxel Density

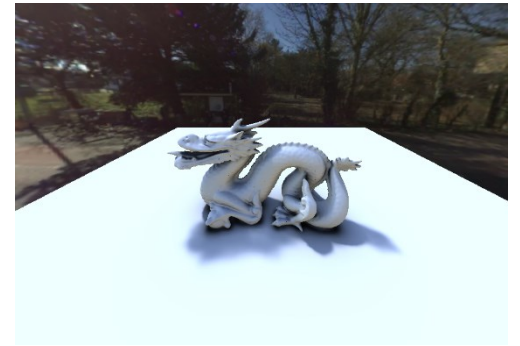
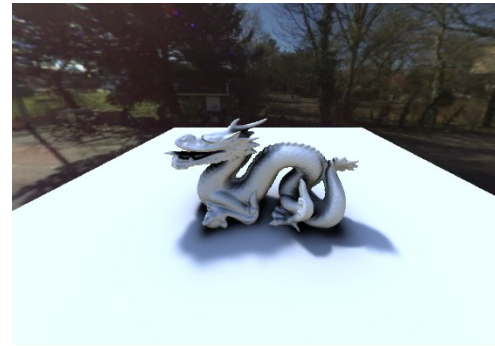
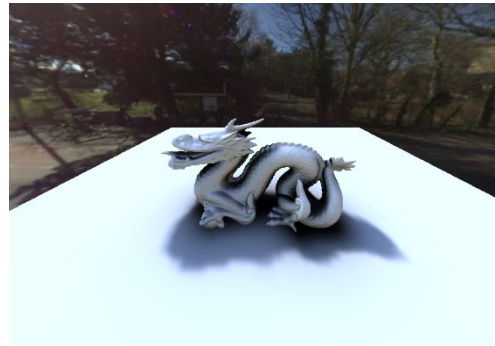
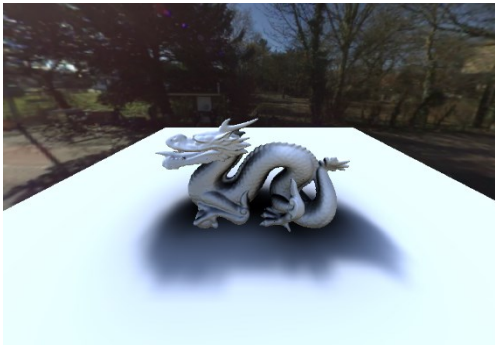


64

increasing voxel volume

512

# Dynamic Visibility – Voxel Density



64

increasing voxel volume

512

# Dynamic Lighting

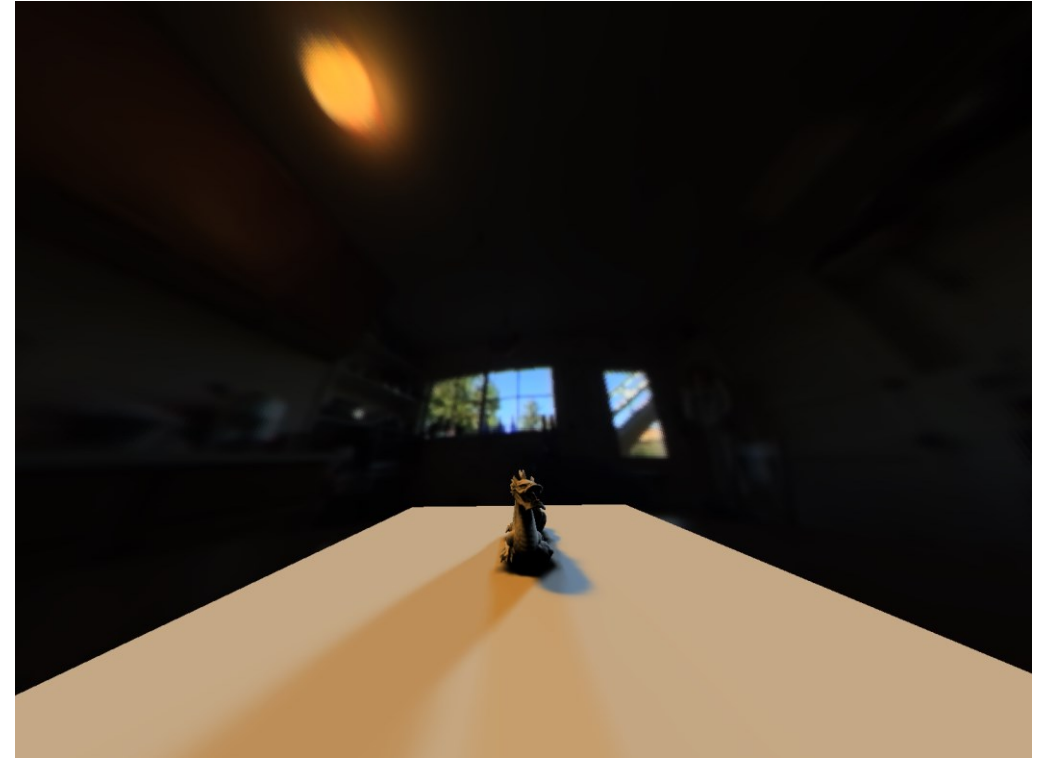


# Peak Detection

---



without peak detection



with peak detection

# Relighting of Real Objects - Inverse Rendering using SRBFs

	Haar wavelets [Haber et al., 2009]	SRBF (ours)
input resolution	960 × 540	960 × 540
# images	5	5
# vertices	2689	2689
# lighting coefficients	1024	1020
# materials	3	3
# BRDF weights	<b>2689 (per vertex)</b>	<b>906240 (per texel)</b>
$\Delta t$ optimize lighting	1089.340 s	10.189 s
$\Delta t$ optimize materials	2.516 s	6.973 s
$\Delta t$ optimize BRDF weights	2.518 s	180.837 s
$\Delta t$ optimize one BRDF weights	<b><math>1.36 \times 10^{-4}</math> s</b>	<b><math>1.99 \times 10^{-4}</math> s</b>
$\Delta t$ one iteration	1099.680 s	201.240 s
$\Delta t$ full optimization	184 min	29 min
$\Delta t$ rendering reconstruction	<b>13.403 s</b>	<b>0.022 s</b>

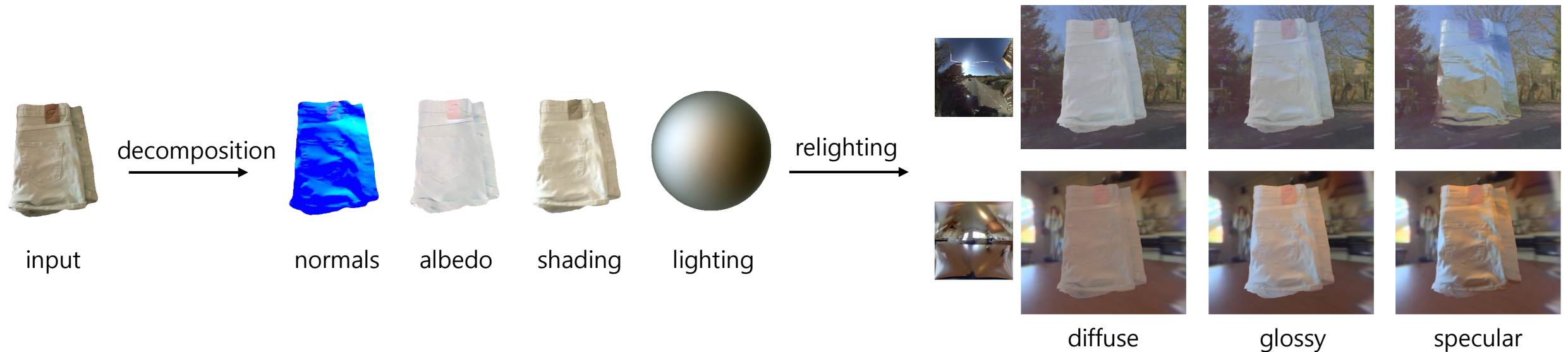


# Relighting of Real Objects – Intrinsic Image Decomposition

- Intrinsic images decomposition using SRBFs [Barron and Malik, 2015]

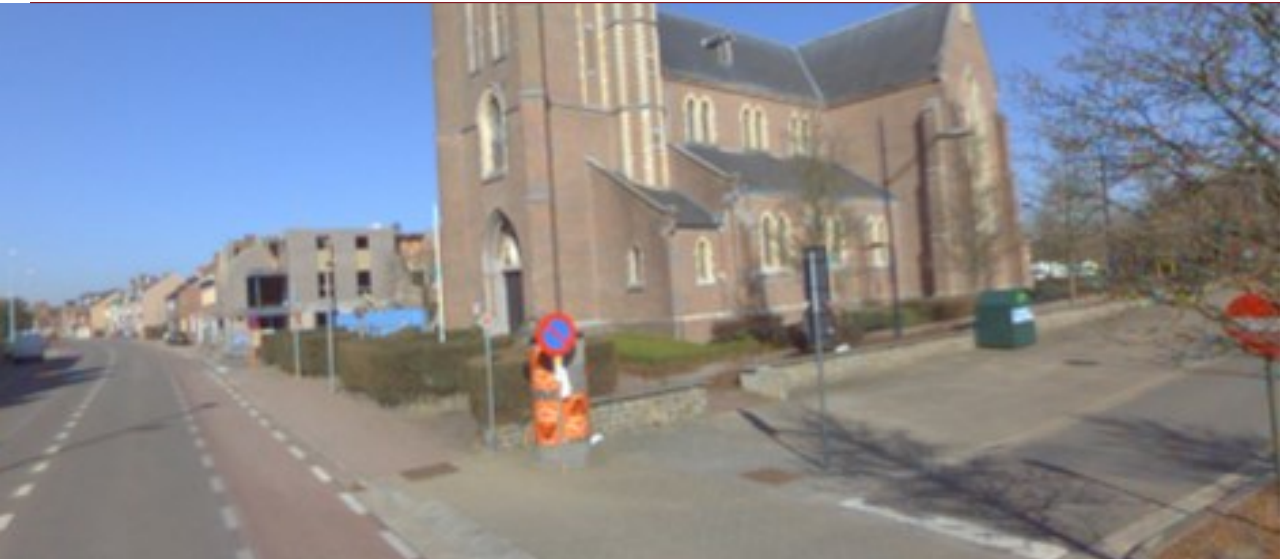
$$\underset{G,R,L}{\text{maximize}} \quad P(G)P(R)P(L)$$

$$\text{subject to} \quad I = R + S(G, L)$$





# Relighting of Virtual Objects [AIVIE, 2014]



# Relighting of Real Objects [ICT-FP7 SCENE, 2014]



## SCENE Relighting