Inverse rendering and relighting applications

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Relighting

\[ B(x, \omega_o) = \int_{\Omega} V(x, \omega_i) \rho(x, \omega_i, \omega_o)(\omega_i \cdot n(x)) L(x, \omega_i) \, d\omega_i \]
Relighting

\[ B(x, \omega_o) = \int_\Omega V(x, \omega_i) \rho(x, \omega_i, \omega_o)(\omega_i \cdot n(x)) L(x, \omega_i) d\omega_i \]
From 2D to the spherical domain

- Octahedron parameterisation
Relighting

\[ B(x, \omega_o) = \int_{\Omega} V(x, \omega_i) \rho(x, \omega_i, \omega_o)(\omega_i \cdot n(x)) L(x, \omega_i) \, d\omega_i \]
Inverse Rendering

- Computer vision <-> computer graphics
  - Reflected light = convolution of lighting and BRDF
  - Inverse rendering = deconvolution

\[ B(x, \omega_o) = \int_{\Omega} V(x, \omega_i) \rho(x, \omega_i, \omega_o)(\omega_i \cdot n(x))L(x, \omega_i) d\omega_i \]
Goal

Relighting Objects from Image Collections, Haber, 2009
Results
Results
Why is it difficult?

• Image <-> array of pixels
Colour constancy
Brightness illusions
Why is it difficult?

• Material-lighting ambiguity
Why is it difficult?

• Problem has a high dimensionality

• Material info is 6D
  • Incident light direction (2D)
  • Viewing direction (2D)
  • Position (2D)
So what can we expect to recover?

• Depends on the assumptions we make (Ramamoorthi, 2001)
  • BRDF known / lighting unknown
  • Lighting known / BRDF unknown
  • Both factors unknown

• Studies various cases
  • Factorised lighting
  • Spherical harmonics used as mathematical tool
  • ~ Fourier series on the sphere
  • Recoverable frequencies proved

• Ringing / Low frequency only

Spherical harmonics
General conclusion

- In order to have high frequencies in the result, you need to have high frequencies in the BRDF and illumination factor.

- Estimating high frequency BRDF requires high frequency lights.

- Estimating high frequency lights requires high frequency BRDF.

- Unsolved problem: solution only unique up to a scale factor.
  - smoothness factor

![Diagram showing the process of combining two images to achieve a desired result.](image-url)
Choosing a good representation

\[ B(x, \omega_o) = \int_{\Omega} V(x, \omega_i) \rho(x, \omega_i, \omega_o) (\omega_i \cdot n(x)) L(x, \omega_i) \, d\omega_i \]

\[
\begin{array}{ccc}
V(x, \omega) &=& \sum_i V_i \Psi_i(\omega) \\
\rho(\omega) &=& \sum_j \rho_j \Psi_j(\omega) \\
L(\omega) &=& \sum_k L_k \Psi_k(\omega)
\end{array}
\]

\[ B(x, \omega_o) = \sum_i \sum_j \sum_k V_i \tilde{L}_j \rho_k C_{ijk} \quad \text{with} \quad C_{ijk} = \int_{\Omega} \Psi_i(\omega_i) \Psi_j(\omega_i) \Psi_k(\omega_i) \, d\omega_i \]
Triple product integral

\[ B = \int_{S^2} L(\omega) V(\omega) \tilde{\rho}(\omega) \, d\omega \]

\[ = \int_{S^2} \left( \sum_i L_i \Psi_i(\omega) \right) \left( \sum_j V_j \Psi_j(\omega) \right) \left( \sum_k \tilde{\rho}_k \Psi_k(\omega) \right) \, d\omega \]

\[ = \sum_i \sum_j \sum_k L_i V_j \tilde{\rho}_k \int_{S^2} \Psi_i(\omega) \Psi_j(\omega) \Psi_k(\omega) \, d\omega \]

\[ = \sum_i \sum_j \sum_k L_i V_j \tilde{\rho}_k C_{ijk} \]

Triple Product Wavelet Integrals for All-Frequency Relighting, Ren Ng, Ravi Ramamoorthi, Pat Hanrahan, SIGGRAPH 2004
Choosing a good representation

• Wavelets
  • 2D-haar
  • Spherical / high-order wavelets

• Spherical Radial basis functions (SRBFs)
  • ~ mixture of Gaussians

• Eigenbases
2D Haar wavelets
Haar wavelet representation

- Compact
  - Localised in spatial and frequency domain

- Supports high frequencies

- Fast triple product (Ng. et al, 2004)
  \[
  B(x, \omega_0) = \sum_i \sum_j \sum_k V_i \tilde{L}_j \rho_k \int_{\Omega} \Psi_i(\omega_i)\Psi_j(\omega_i)\Psi_k(\omega_i) d\omega_i
  \]

- Reasonably fast rotation method (Wang et al, 2006)
What is \( \tilde{L} \)?

- Rotated version of lighting
- Wavelet coefficients rotated by precalculated sparse rotation matrices
- \( O(N) \), \( N = \) number of sparse wavelet coefficients
Precalculated rotation matrices
High-level algorithm overview

- input images
- geometry

environment map per viewpoint

optimization

materials per vertex

Estimation of Haar wavelet coefficients

rerendered using extracted appearance
Problem statement

\[ L(\tilde{x}, w_0) = \sum_k \sum_l \sum_m C_{klm} \hat{\rho}_k V_l \tilde{L}_m \]  

\text{(Discrete triple product integral)}

\[ C_{klm} = \int_{\Omega} \Psi_k \Psi_l \Psi_m d\omega \]  

\text{(Triple product binding coefficients)}

\[ T_{x, km} = \sum_l C_{klm} V_l \]  

\text{(Transfer function)}

\forall y_p : L_p = \rho \rightarrow T \rightarrow \tilde{L} \]  

\text{(Bilinear problem in matrix notation)}

\[ O = \sum_{p=1}^{N} \alpha_p \left(y_p - \rho \rightarrow T \rightarrow \tilde{L}\right)^2 \]  

\text{(Optimisation objective function)}
How do we solve the problem?

- Fundamentally underconstrained!
- Approximate best solution with Quadratic Optimisation
  - Fast primal-dual interior point solver: OOQP (Gertz, 2003)

\[
O = \sum_{p=1}^{N} \alpha_p \left( y_p - \rho \frac{1}{T} \tilde{L} \right)^2
\]

\[
\min_x \| Mx - Y \|_2
\]

\[
\min_x \frac{1}{2} x^T Q x + c^T x
\]

\[
A x = b, \quad C x \geq d
\]

\[
M \in \mathbb{R}^{\text{pixels} \times \text{coeffs}}
\]

\[
Q \in \mathbb{R}^{\text{coeffs} \times \text{coeffs}}
\quad Q = M^T M
\]

\[
c \in \mathbb{R}^{\text{coeffs}}
\quad c = M^T Y
\]

\[
x \in \mathbb{R}^{\text{coeffs} \times 1}
\quad Y \in \mathbb{R}^{\text{pixels} \times 1}
\]

\[
\text{Not used}
\]

}\[
\text{Pixels positive}
\]
Solution conditions

• The Quadratic problem is convex and has a unique global optimum
  • Can be proved

• However, local minimum still possible in bilinear problem
  • Alternating between L and $\rho$ suboptimal
Trade-offs

• Static lighting vs varying lighting

• Single- vs multi-view

• Haber et al. [HFB+09]
  • All frequency wavelet framework
  • Incident illumination per image
  • Reflectance per surface point
Future work

• Support for local lighting
  • Most techniques require light at infinity
  • Hard to estimate lights inside the scene
  • Different lighting information per pixel
    = too much data!

• Support for indirect lighting
  • Currently ignored in equations
Papers to read

• A signal processing framework for inverse rendering, Ramamoorthi, 2001

• Relighting objects from image collections, Haber et al, 2009

Expect high-level questions about the algorithms/processes on exam
Questions?